Industrial Organization

Final Exam Spring 2024 – Solution Topics

1. False.

The equilibrium price resulting from the static Bertrand model of two firms with equal total cost functions and homogenous products will be equal to their marginal cost, due to the constant incentive to undercut their competition until they arrive at P = MC (i.e., the Bertrand Paradox). However, under an infinitely-repeated game, if the discount factor is less than ½, collusion will not be sustained, hence leading to an equilibrium price equal to that of the static game. Therefore, the statement is false.

2. True.

Assuming firms are tacitly colluding in prices, facing a decrease in the marginal and average cost of production, firms may fear decreasing price as it may be interpreted by the other firms as a deviation from their collusive agreement, potentially leading to a price war. Thus, they may choose to remain with their price unaltered despite decrease in costs due to this fear. Hence, the statement is true.

3.

(i)

Firms will collude as long as the present value of profits under collusion is higher than the present value of profits under deviation:

$$\frac{\pi^M}{2} + \delta\frac{\pi^M}{2} + \delta^2\frac{\pi^M}{2} + \cdots \geq \pi^M \Leftrightarrow \cdots \Leftrightarrow \delta \geq \frac{1}{2}$$

A discount factor greater than or equal to ½ is required to sustain collusion under this scenario.

(N.B. complete workings out are required for full marks.)

(ii)

The optimal collusion price will be the price that maximizes total profits, in other words, the monopoly price:

Monopoly's profit-maximization problem:

$$\max_p \pi = p * q(p) - 2q(p)$$

$$FOC: \frac{d\pi}{dp} = 0 \Leftrightarrow 10 - 2p + 2 = 0 \Leftrightarrow p = 6$$

The optimal tacit collusion price is equal to 6, this is where the sum of their profits will be highest.

(iii)

Assuming that all firms collude at the optimal tacit collusion price of 6, the total quantity of 4 will be split among the 3 firms, meaning each will produce 4/3.

Firm C's profit under collusion at $p = 6$:

$$\pi_C = (6 - 5) * \frac{4}{3} = \frac{4}{3}$$

If Firm C decides to deviate in the first period in will charge a price slightly below 6:

$$p_c = 6 - \varepsilon$$
$$\varepsilon \to 0$$
$$\pi_C = (6 - 5) * 4 = 4$$

From then onwards Firm C will no longer make any profit.

Hence to sustain collusion for Firm C:

$$\frac{4}{3} + \delta\frac{4}{3} + \delta^2\frac{4}{3} + \cdots \geq 4 \Leftrightarrow \cdots \Leftrightarrow \delta \geq \frac{2}{3}$$

Firm C is able to tacitly collude around the price of 6 for discount factors greater than or equal to 2/3.

(N.B if you use monopoly profits instead of 4 in the equation, the same value for the discount factor is found. However, this is a coincidence and incorrect nonetheless.)

(iv)

For Firms A and B to sustain collusion:

$$\frac{\pi^M}{3} + \delta\frac{\pi^M}{3} + \delta^2\frac{\pi^M}{3} + \cdots \geq \pi^M \Leftrightarrow \cdots \Leftrightarrow \delta \geq \frac{2}{3}$$

Thus, Firm C's entry has made it harder for tacit collusion to emerge in this market as the required discount factor in the market has increased.

(v)

Under the previous tacit collusion agreement, the association's members (Firms A and B) would produce 2/3 of the total quantity, implying a total profit for them of 32/3.

If Firms A and B were to charge a price of slightly below 5:

$$p = 5$$

$$q = 5$$

$$\pi = (5 - 2) * 5 = 15$$

Hence, under this new recommended price the association has a higher profit (15>32/3), and that is why the director-general proposed it.

4.

(i)

The optimal collusion quantity will be the quantity that maximizes total profits, in other words, the monopoly quantity:

Monopoly's profit-maximization problem:

$$\max_q \pi \ = p(q) * q - 2q$$

$$FOC: \frac{d\pi}{dq} = 0 \Leftrightarrow 10 - 2q - 2 = 0 \Leftrightarrow \ q = 4$$

$$q_1 = q_2 = 2$$

$$p = 10 - 2 - 2 = 6$$

$$\pi_1 = \pi_2 = (6 - 2) * 2 = 8$$

(ii)

Firms will collude as long as the present value of profits under collusion is higher than the present value of profits under deviation.

Under collusion they will each have a profit equal to 8.

Assuming deviation, one firm will engage their best response to the other firm producing half the monopoly quantity (i.e., $q_2 = 2$):

Firm 1's profit-maximization problem:

$$\max_{q_1} \pi_1 = P(q_1, q_2)q_1 - 2q_1$$

$$FOC: \frac{d\pi_1}{dq_1} = 0 \Leftrightarrow 10 - 2q_1 - q_2 - 2 = 0 \Leftrightarrow q_1^* = 4 - \frac{q_2}{2}$$

$$q_1^* = 4 - \frac{2}{2} = 3$$

$$p = 10 - 3 - 2 = 5$$

$$\pi_1 = (5 - 2) * 3 = 9$$

From then onwards they would engage in competition *à la* Cournot (in equilibrium, $q_1 = q_2$):

$$q_1 = 4 - \frac{q_2}{2}$$

$q_1 = q_2$ (in equilibrium)

$$q_1 = q_2 = \frac{8}{3}$$

$$p_1 = p_2 = \frac{14}{3}$$

$$\pi_1 = \pi_2 = \left(\frac{14}{3} - 2\right) * \frac{8}{3} = \frac{64}{9}$$

Therefore, in order for the two firms to tacitly collude:

$$8 + 8\delta + 8\delta^2 + \cdots \geq 9 + \frac{64}{9}\delta + \frac{64}{9}\delta^2 \Leftrightarrow \cdots \Leftrightarrow \delta \geq \frac{9}{17}$$

A discount factor greater than or equal to 9/17 is required to sustain collusion in this case.

(iii)

For Firm 1:

$$r_1 < r \Rightarrow \delta_1^{New} > \delta_1^{Old}$$

For Firm 2:

$$r_2 > r \Rightarrow \delta_2^{New} < \delta_2^{Old}$$

Firm 1's discount factor will be larger than before whilst for Firm 2, its discount factor will be smaller than before. For collusion to occur in this market, the discount factors of both firms need to be greater than 9/17. Since now one of the firms has a lower discount factor, it is less likely that a collusive agreement would occur.

(iv)

Firm 2 is responsible for the decrease in likelihood of tacit collusion since it registered a fall in its discount factor from the rise of its interest rate, making it less likely to surpass the required threshold of 9/17.