

**Industrial Organization**  
**Final Exam Spring 2023 – Solution Topics**

1. False: Mary is right.

Suppose that, for a given demand and marginal cost, the dominant firm's optimal choice involves a price below the fringe supply curve. In practice, that would mean that firms in the competitive fringe are incredibly inefficient – even their minimum possible marginal cost is above the market price – so they would not sell any units.

2. False.

If firms do not discount future payoffs ( $\delta = 1$ )<sup>1</sup> and the game is infinitely repeated, firms will be inclined to engage in a collusive agreement. However, this does not necessarily mean that the collusive price will significantly exceed firms' marginal costs. In fact, even though the optimal collusive price is the monopoly price, collusion can occur at any price point above the marginal cost, implying that collusion can be observed even when the price is close to the marginal cost ( $P = MC + \varepsilon$ ).

3.

(i)

Firms compete *à la* Stackelberg if firm *I* (Leader) chooses to accommodate the entry of firm *E* (Follower). Therefore, we must start by solving firm *E*'s profit-maximization problem, in order to find the Follower's best-response function:

$$\begin{aligned}\max_{q_E} \pi_E &= (10 - q_I - q_E)q_E - 4q_E \\ \text{FOC: } \frac{\partial \pi_E}{\partial q_E} &= 0 \Leftrightarrow 10 - q_I - 2q_E - 4 = 0 \Leftrightarrow q_E = 3 - \frac{q_I}{2}\end{aligned}$$

Then, we must incorporate firm *E*'s best-reply function in firm *I*'s profit-maximization problem, so as to find the Leader's optimal choice of quantity:

$$\begin{aligned}\max_{q_I} \pi_I &= (10 - q_I - q_E)q_I - 3q_I \Leftrightarrow \max_{q_I} \pi_I = \left(10 - q_I - 3 + \frac{q_I}{2}\right)q_I - 3q_I \\ \text{s.t. } q_E &= 3 - \frac{q_I}{2} \\ \text{FOC: } \frac{\partial \pi_I}{\partial q_I} &= 0 \Leftrightarrow 10 - 2q_I - 3 + q_I - 3 = 0 \Leftrightarrow q_I = 4\end{aligned}$$

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<sup>1</sup>  $\delta = 1$  implies that firms value future profits as much as current profits.

Finally, we may plug the Leader's optimal choice into the Follower's BR function:

$$q_E(q_I = 4) = 3 - \frac{4}{2} = 1 \rightarrow Q = 5 \rightarrow P = 5$$

Therefore,  $\pi_I = (5 - 3) * 4 = 8$  and  $\pi_E = (5 - 4) * 1 = 1$

(ii)

If firm  $I$  chooses to deter firm  $E$ 's entry, then it will have to produce a quantity such that firm  $E$  would be unprofitable (i.e., its optimal choice would be not to produce):

$$q_E \leq 0 \rightarrow 3 - \frac{q_I}{2} \leq 0 \rightarrow q_I \geq 6$$

Given that the monopoly quantity is  $q_I^M = 3.5$ , it is clear that the optimal choice of quantity under a price limit strategy is  $q_I = 6$ . Thus,  $P = 4$  and  $\pi_I = (4 - 3) * 6 = 6$  ( $\pi_E = 0$ , naturally).

(iii)

No, firm  $I$  would not be a monopolist: while it would be the single producer in this market, it would not have the ability to earn the monopoly profit. [Notice that, if firm  $I$  were to produce the monopoly quantity, firm  $E$ 's best response would be to *enter* the market!]

(iv)

Since profits are higher when firm  $I$  accommodates  $E$ 's entry ( $\pi_I^{(i)} = 8 > 6 = \pi_I^{(ii)}$ ), that will be the chosen strategy.

(v)

We now have competition in prices with homogeneous products. Even though we are assuming that choice is sequential, the rationale behind Bertrand competition still applies:

- If firm  $I$  were to choose any price **above** the entrant's marginal cost of 4, firm  $E$  would undercut and be the sole firm in the market.
- If firm  $I$  were to choose a price **equal to** the entrant's marginal cost of 4, they would split the market.
- If firm  $I$  were to **undercut** the lowest price firm  $E$  may quote (4), thus charging  $P_1 = 4 - \varepsilon$ , they would be the only firm left in the market.

It is clear that the most favorable choice for firm  $I$  is the third option. Considering  $\varepsilon \rightarrow 0$ ,  $\pi_I = (4 - 3) * 6 = 6$  ( $\pi_E = 0$ , naturally).

(vi)

Competition in prices leads to lower prices ( $P^{(i)} = 5 > 4 = P^{(v)}$ ). Hence, it is preferred by consumers.

4.<sup>2</sup>

(i)

Firms will collude as long as the present value of profits under collusion is higher than the present value of profits under deviation:

$$\frac{\pi^M}{2} + \delta \frac{\pi^M}{2} + \delta^2 \frac{\pi^M}{2} + \dots \geq \pi^M \Leftrightarrow \dots \Leftrightarrow \delta \geq \frac{1}{2}$$

The optimal collusive price is determined by the consumer valuation, which is set at 10. When firms collude around this optimal price, each firm sells to 5 consumers, resulting in a profit of  $\pi_i = (10 - 2) \times 5 = 40$ .

(ii)

As before, firms will collude as long as the present value of profits under collusion is higher than the present value of profits under deviation. In this case, profits under collusion increase by 4 each year. Hence, firms will choose to collude if the following condition is met:

$$40 + 44\delta + 48\delta^2 + \dots > 80$$

Previously, when the market did not experience any growth over time, firms would collude if:

$$40 + 40\delta + 40\delta^2 + \dots > 80$$

By comparing these two situations, we can conclude that the present value of profits under collusion has increased, while the present value of profits under deviation remains constant. Consequently, the minimum discount factor,  $\delta$ , required for collusion to be preferred over deviation, becomes smaller. This implies that sustaining a collusive agreement has become relatively easier.

In summary, a growing demand facilitates tacit collusion.

(iii)

As before, the optimal collusive price is the valuation of each consumer i.e., 10.

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<sup>2</sup>You needed to show all the relevant intermediate steps in part (i), not just jump from the initial inequality to the conclusion!

(iv)

Applying the same reasoning as in question (ii), when compared to the value calculated in (i), a shrinking demand over time implies a decrease in the present value of profits under collusion, while the present value of profits under deviation remains constant:

$$40 + 36\delta + \dots + 0 > 80$$

As a result, the minimum discount factor,  $\delta$ , necessary for collusion to be favored over deviation, becomes larger.

Therefore, a shrinking demand hinders tacit collusion.

(v)

The diminishing demand from one consumer each year marks a transition in the dynamics of interaction between these two firms from infinite to finite (this interaction will now span a duration of 10 periods).

Employing the concept of backward induction, the game unfolds from the end to the beginning leading to a solution of  $P = MC = 2$  for each period.

On the final period, both firms will lack incentives to cooperate and will instead resort to charging  $P = MC$ , as there are no consequences afterwards. This same rationale holds true for all preceding periods.

In essence, the finite nature of the firms' interaction eliminates any possibility of cooperation, resulting in both firms charging  $P = MC = 2$  throughout all periods.