Industrial Organization

Midterm Spring 2023 – Solution Topics

1. True.

Consider a symmetric Bertrand duopoly in which firms have a constant marginal and average cost of c. In equilibrium, $P_1 = P_2 = MC_1 = MC_2 = c$. Suppose now that a third firm enters the market with $MC_3 = c$. This firm's entry will have absolutely no effect on consumer welfare, given that the market price will still be equal to c (and Demand has not changed, so $Q^{Bertrand}$ will also be the same). Therefore, it is proven that there are exceptions to this 'rule'.

2. False.

Consider a symmetric Bertrand duopoly in which firms have a constant marginal and average cost of c. In the initial equilibrium, both firms charge the same price ($P_1 = P_2 = MC_1 = MC_2 = c$) and – by assumption – produce the same quantity ($q_1 = q_2$), resulting in an HHI of 0,5 ($HHI = \sum_{i=1}^{2} s_i^2 = 0{,}5$).
Suppose that firm 1 discovers a new technology that reduces its marginal cost to $MC_1' < MC_1$. After this technological improvement, firm 1 lowers its price to $P_1 = MC_2 - \varepsilon$, while firm 2 keeps its price unchanged. As $P_1 < P_2$ firm 1 becomes the only firm producing a positive amount of output in this market, implying that $s_1 = 100\%$ while $s_2 = 0\%$.
Under the new equilibrium, the HHI increased from 0,5 to 1 while the equilibrium price decreased.

3.

(i)

Firm 1's profit-maximization problem:

$$\max_{q_1} P(q_1 + q_2)q_1 - 2q_1$$

$$FOC: \frac{d\pi_1}{dq_1} = 0 \Leftrightarrow 10 - 2q_1 - q_2 - 2 = 0 \Leftrightarrow \boldsymbol{q_1^* = 4 - \frac{q_2}{2}}$$

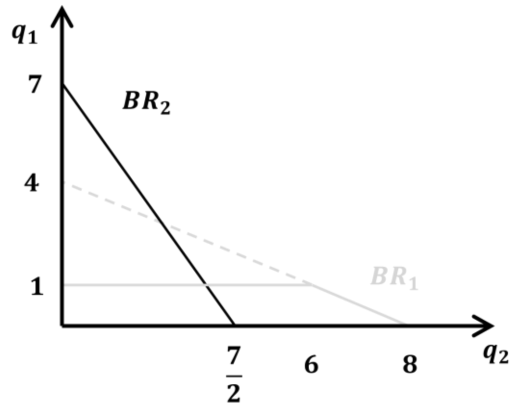
Due to the capacity constraint of firm 1, its complete Best Response is given by the following function:

$$BR_1: \begin{cases} q_1 = 4 - \frac{q_2}{2} \; if \; q_2 \geq 6 \\ q_1 = 1 \; if \; q_2 < 6 \end{cases}$$

Firm 2's profit-maximization problem:

$$\max_{q_2} P(q_1 + q_2)q_2 - 3q_2$$

$$FOC: \frac{d\pi_1}{dq_2} = 0 \Leftrightarrow 10 - 2q_2 - q_1 - 3 = 0 \Leftrightarrow \boldsymbol{q_2^*} = \frac{\mathbf{7}}{\mathbf{2}} - \frac{\boldsymbol{q_1}}{\mathbf{2}}$$



(ii)

To find the equilibrium, one must find the intersection between the Best Response functions of the two firms present in the market.

$\begin{cases} q_1 = 4 - \frac{q_2}{2} \\ q_2 = \frac{7}{2} - \frac{q_1}{2} \end{cases} \Leftrightarrow \begin{cases} q_1 = 3 \\ q_2 = 2 \end{cases} \rightarrow$ Impossible as firm 1 has a capacity constraint of 1 physical unit.

Given this, $q_1 = 1 \rightarrow q_2 = \frac{7}{2} - \frac{1}{2} = 3$. As $Q = q_1 + q_2 = 4 \rightarrow P^*(Q = 4) = 10 - 4 = 6$

$$\pi_1 = (6 - 2) * 1 = 4$$

$$\pi_2 = (6 - 3) * 3 = 9$$

(iii)

With the new machinery firm 1 would not face a capacity constraint. Consequently, the equilibrium would be:

$$\begin{cases} q_1 = 4 - \frac{q_2}{2} \\ q_2 = \frac{7}{2} - \frac{q_1}{2} \end{cases} \Leftrightarrow \begin{cases} q_1 = 3 \\ q_2 = 2 \end{cases} \rightarrow P^*(Q = 5) = 5$$

$$\pi_1 = (5 - 2) * 3 = 9$$

$$\pi_2 = (5 - 3) * 2 = 4$$

Firm 1 would be willing to pay up to 5 for the new machinery ($\Delta\pi = 9 - 4 = 5$).

(iv)

The installation of the new machinery will be socially beneficial as long as total welfare increases.

Initially, given the market equilibrium of $P^* = 6 \wedge Q^* = 4$:

$$CS = 8 \wedge PS = 9 + 4 = 13 \rightarrow TW = 13 + 8 = 21.$$

With the adoption of the new machinery, given the results obtained in (iii), the market equilibrium is $P^* = 5 \wedge Q^* = 5$. Therefore:

$$CS = 12{,}5 \wedge PS = 9 - F + 4 = 13 - F \rightarrow TW = 25{,}5 - F$$[1]

This implies that the adoption of this new machinery will be socially beneficial as long as $F < 4{,}5$.

(v)

Yes, by enhancing its competitive ability vis-à-vis firm 2, the adoption of this new technology by firm 1 generates 2 externalities: (i) a decrease in the profit of firm 2 – negative externality, and (ii) an increase in consumer surplus – positive externality.

4.

(i)

$\frac{\partial q_1}{\partial P_2} = \frac{\partial q_2}{\partial P_1} = -1 < 0 \Rightarrow$ the goods are <u>complements</u>, because the quantity demand of each good decreases as a result of an increase in the other good's price.

(ii)

Firm 1's profit-maximization problem:

$$\max_{\{P_1\}} \pi_1 = P_1(10 - P_1 - P_2) - 2(10 - P_1 - P_2)$$

$$FOC: \frac{d\pi_1}{dP_1} = 0 \Leftrightarrow 10 - 2P_1 - P_2 + 2 = 0 \Leftrightarrow \boldsymbol{P_1^* = 6 - \frac{P_2}{2}}$$

Firm 2's demand is symmetric to Firm 1's. Besides, Firm 2's cost structure (in particular, the marginal cost) is equal to Firm 1's. Therefore, in equilibrium, $\boldsymbol{P_1^* = P_2^*}$ (symmetry).

$$\begin{cases} P_1 = 6 - \frac{P_2}{2} \\ P_1^* = P_2^* \end{cases} \Leftrightarrow \begin{cases} \frac{3P_1}{2} = 6 \\ \_ \end{cases} \Leftrightarrow \begin{cases} \boldsymbol{P_1^* = 4} \\ \boldsymbol{P_2^* = 4} \end{cases}$$

$$q_1 = 10 - 4 - 4 = 2; q_2 = 2$$

$$\boldsymbol{\pi_1 = (4 - 2) * 2 = 4; \pi_2 = 4}$$

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[1] F represents the cost to implement the new machinery.

(iii)

Notice the negative sign in Firm 1's best-reply function (and, equivalently, in 2's BR):
$\frac{\partial \boldsymbol{P_1^*}}{\partial P_2} = \frac{\partial \boldsymbol{P_2^*}}{\partial P_1} = -\frac{1}{2} < 0 \Rightarrow$ the firm's decision variables are <u>strategic substitutes</u> because the optimal response to an increase in the other firm's price is decreasing one's own.

(iv)

It is clear, from **(iii)**, that there is a negative externality between firms 1 and 2: each firm's decision to charge a higher price has a negative impact on the quantity demanded not just for its own product, but also for *the other firm's product*.

(v)

Firm 1's new profit-maximization problem (single firm producing both goods):

$$\max_{\{P_1,P_2\}} \pi_1 = P_1(10 - P_1 - P_2) + P_2(10 - P_2 - P_1) - 2(10 - P_1 - P_2) - 2(10 - P_2 - P_1)$$

$$FOC: \frac{d\pi_1}{dP_1} = 0 \Leftrightarrow 10 - 2P_1 - P_2 - P_2 + 4 = 0 \Leftrightarrow \boldsymbol{P_1 = 7 - P_2}$$

If you took the second first-order condition $\left(\frac{d\pi_1}{dP_2} = 0\right)$, the result would be $\boldsymbol{P_2} = \boldsymbol{7 - P_1}$...

In practice, this means that optimal prices are defined by the condition $\boldsymbol{P_1 + P_2} = \boldsymbol{7}$.[2]

Thus,

$$q_1 = 10 - (\boldsymbol{P_1 + P_2}) = 10 - 7 = 3; q_2 = 3$$

$$\boldsymbol{\pi_1} = \boldsymbol{P_1} * 3 + \boldsymbol{P_2} * 3 - 2 * 2 * 3 = (\boldsymbol{P_1 + P_2}) * 3 - 12 = 7 * 3 - 12 = \boldsymbol{9}$$

(vi)

$$\boldsymbol{P_1^{(v)} + P_2^{(v)} = 7 < 8 = P_1^{(ii)} + P_2^{(ii)}}$$

When firms are merged, the externality identified in **(iv)** is internalized – the prices found in **(v)** maximize *overall* profits (i.e., the externality is "incorporated" in the decision problem). Meanwhile, in **(ii)**, the prices that you found were such that *individual* profits were maximized.

[2]Some of you assumed symmetry (i.e., $P_1 = P_2 = 3.5$). This is just one of the infinite possible price combinations that are optimal (i.e., those that respect the condition $P_1 + P_2 = 7$). Besides, the assumption was not necessary to solve the exercise, but no points were taken from your grade as a result of this simplification.