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Bertrand without fudge

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Abstract

This paper reexamines Bertrand competition with homogeneous products and different marginal costs. It is shown that the conventional outcome is supported by an equilibrium in the original game under the standard rationing rule.

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It is a common exercise in textbooks for microeconomics and industrial organization, both at the graduate and undergraduate level, to ask students to derive an equilibrium for Bertrand competition between two firms, who produce homogeneous products and have different marginal costs. Given the simplicity of the question it may be a surprise to the newcomer and a slight embarrassment to the profession that the answers that are typically given are awkward, inexact and sometimes wrong.

One solution that is often offered involves discretizing the strategy space. Another closely related solution is to alter the solution concept to limits of ϵ-equilibria. Sometimes, there is an appeal to a rationing rule that favors the low-cost firm. Sometimes it is asserted that there is no solution, unless one relies on one of these methods. The point of this note is to show that the latter claim is false and that the common practice of fudging the issue is both unnecessary and misleading. The conventional outcome, i.e. the low-cost firm charges a price equal to the high-cost firm's cost, is supported by an equilibrium in the original game under the standard rationing rule that both firms split the market if their prices coincide. Moreover, any equilibrium that supports this outcome has the appealing property that it does not rely on the use of dominated strategies.

To fix ideas, consider two firms with commonly known constant marginal costs c_1 and c_2 respectively, where $c_1 < c_2$. Both firms produce homogeneous products, face a strictly decreasing

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differentiable market demand function $D(p)$ on $[0,\bar{p}]$ with $c_2 < \bar{p} \leq \infty$, and the price the low-cost firm would charge if it were a monopoly, $p^m(c_1)$, satisfies $c_2 < p^m(c_1)$.

We claim that for small enough $\eta > 0$, the following is an equilibrium: The low-cost firm posts a price equal to c_2. The high-cost firm randomizes uniformly over $[c_2, c_2 + \eta]$; denote this distribution by $F(p;\eta)$ and its density by $f(p;\eta)$. It is trivial to check that the high-cost firm's strategy is a best response; it earns zero profits, the best it can do, given firm one's strategy. It is also clear that the low-cost firm does not prefer to post prices outside of the support of firm two's mixed strategy. It remains to check that firm one does not prefer posting a price in $(c_2, c_2 + \eta]$. For this it suffices to show that the derivative of firm one's profit, $\pi_1(p) = D(p)(p - c_1)(1 - F(p;\eta))$, is negative for all $p \in (c_2, c_2 + \eta]$. This derivative equals

$$-f(p;\eta)D(p)(p - c_1) + (1 - F(p;\eta))(D'(p)(p - c_1) + D(p)).$$

The claim follows because $D(c_2)$ is finite since $D(p)$ is decreasing, $D(c_2 + \eta)$ is positive and bounded below for small enough η, and $f(p;\eta)$ goes to infinity for small enough η.

As a technical detail, note that there is a continuum of similar strategies that support any price $p \in (c_1, c_2]$ as a Nash equilibrium price. Among those however, $p = c_2$ is the only price supported by an equilibrium in undominated strategies.

To conclude, it turns out that the case of different marginal costs is much better behaved than the non-generic case of identical marginal costs, where typically the only equilibrium requires firms to use dominated strategies.