Industrial Organization

Bertrand Model

Bertrand with Differentiated Products

Week 7

The Bertrand Model

EXAMPLES: telecommunications, automobile industry, tech industry, etc.

	Perfect competition	Monopolistic competition	Oligopoly	Monopoly
Characteristics	Homogeneous product Many firms Many consumers Free entry and exit	Many firms Many consumers Differentiated product Free entry and exit ($\pi^{LR} = 0$)	**Some** producers but not many Depending on the characteristics of the oligopoly we will use different models. Examples: → Cournot; → Stackelberg; → **Bertrand**; → …	Only one firm Unique product Entry barriers
Behaviour	Takes market determined price as given and chooses quantity to maximize profits ("Price-takers")	Sets price and quantity to maximize profits ("Price-maker")		Sets price and quantity to maximize profits ("Price-maker")
Optimal decision	$P = MC$	$MR = MC$		$MR = MC$

The Bertrand Model

Main assumptions:

- Oligopolistic model (more than one firm)
- Firms choose **simultaneously**
- … the **price they will charge**
- … for a **homogenous product**

Additional assumptions:

- No capacity constraints.
- If $p_1 = \cdots = p_n \rightarrow q_1 = \cdots = q_n = \frac{Q}{n}$

⚠ Not mandatory! Just for simplicity!

Bertrand Equilibrium

- Take a duopoly in which both firms have a Marginal Cost of 10 and demand is given by $P = 100 - Q$. If firms quote prices simultaneously and independently, what will be the market price(s)? [**Hint:** start by assuming that firm 2 would charge the monopoly price ($P^M = 55$). What would firm 1 do?]

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Bertrand Equilibrium

- Each firm has incentives to undercut each other until they reach the marginal cost.

$$MC_1 = MC_2 = c \rightarrow P_1 = P_2 = c$$ → **Bertrand Paradox**

The Bertrand Model

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Bertrand Equilibrium

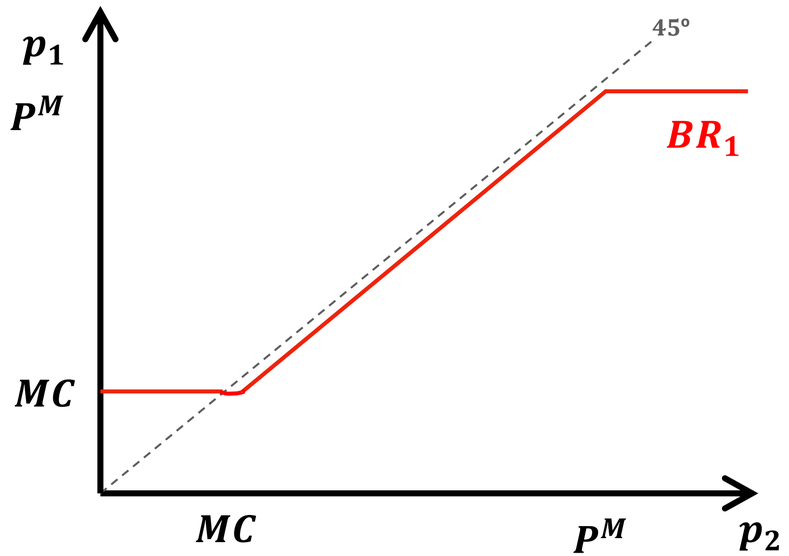
- Would the conclusions of the previous exercise change if firm 1 has $MC_1 = 10$ and firm 2 has $MC_2 = 11$?

The Bertrand Model

Bertrand Equilibrium/Paradox (when both firms have the same cost structure)

- Each firm has incentives to undercut each other until they reach the marginal cost.

$$BR_1: P_1^*(P_2) = \begin{cases} P^M \ if \ P_2 > P^M \\ P_2 - \varepsilon \ if \ MC \leq P_2 \leq P^M \\ MC \ if \ P_2 < MC \end{cases}$$



The Bertrand Model

Bertrand Equilibrium/Paradox (when both firms have the same cost structure)

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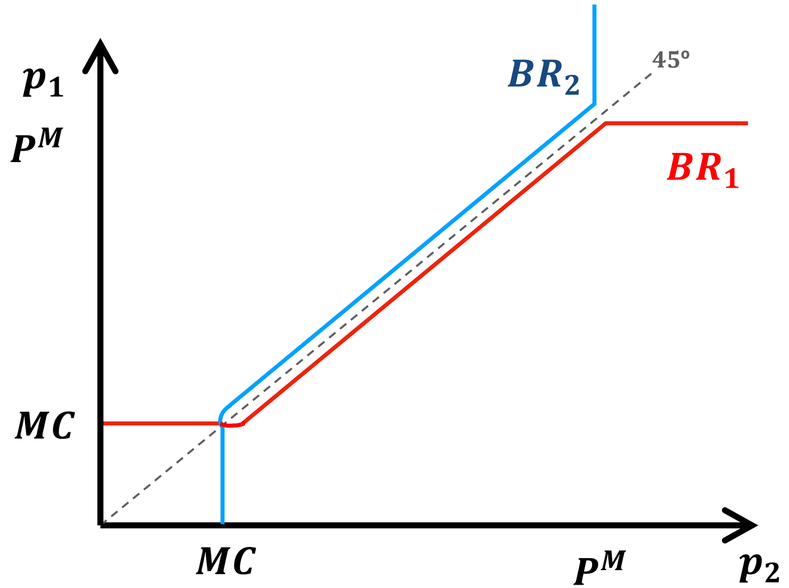

The Bertrand Model

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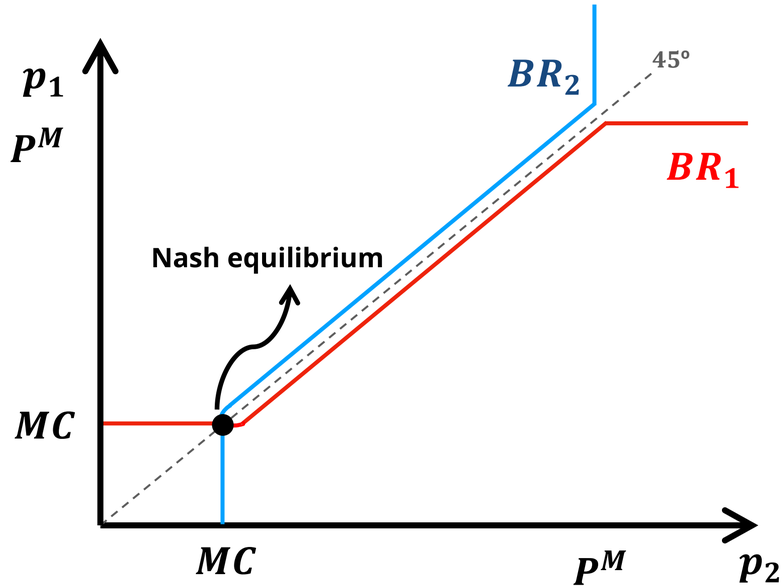
$MC_1 = MC_2 = c \rightarrow P_1 = P_2 = c$ → **Bertrand Paradox**



The Bertrand Model

EXERCISE

8. [ADAPTED] Two firms, an old one, denoted O, and a new one, denoted N, compete in prices, which they set simultaneously and independently while serving the market for an homogeneous product whose demand equals $Q = 10 - P$.

Both firms have a constant marginal and average cost of 2. This information is common knowledge.

(a) What is the market equilibrium price?

Assume now that the new firm's constant marginal and average cost is only known to itself. On the other hand, the old firm knows that firm N produces at a constant marginal and average cost, which can equal either 3 or 1. Firm O attaches equal probability, 0.5, to firm N's two possible marginal and average costs. This incomplete but accurate information held by the old firm is common knowledge.

(b) What price should the old firm choose? Quantify and explain.

(c) What price should the new firm choose? Quantify and explain.

The Bertrand Model

EXERCISE

8. (Continuation)

(d) What is the old firm's expected profit? Quantify and explain.

(e) What are the new firm's possible profits? Quantify and explain.

(f) Would it be socially better if firm N's constant marginal and average cost was common knowledge? Quantify and explain.

Revision

A side-by-side comparison of the Cournot and Bertrand models:

Cournot Model	**Bertrand Model**
✓ Firms **choose quantities** (decision variable: q_i)	✓ Firms **choose prices** (decision variable: P_i)
✓ **Simultaneous** choice	✓ **Simultaneous** choice
✓ Homogeneous product	✓ Homogeneous product

Note: the equal distribution of quantities in a Bertrand exercise in which all firms have the same cost structure (e.g., 50/50 division in a symmetric Bertrand duopoly) is an assumption!

Betrand with differentiated products

COMPETITION IN PRICES WITH DIFFERENTIATED PRODUCTS

Main assumptions:

- Oligopolistic model (more than one firm)
- Firms choose **simultaneously**
- … the **price they will charge** (in the profit-maximization problem, p_i will be the decision variable)
- … of a **differentiated product**

Betrand with differentiated products

THE CASE OF TWO SYMMETRIC FIRMS

General case → $q_1 = a - bp_1 + dp_2 \wedge q_2 = a - bp_2 + dp_1 \wedge MC_1 = MC_2 = c$

Goal of all firms → Maximize profits

$$\max_{p_1} \pi_1 = (P_1 - c)q_1$$

$$\frac{d\pi}{dp_1} = 0 \leftrightarrow (\dots) \leftrightarrow p_1 = \frac{a + cb}{2b} + \frac{d}{2b}p_2$$

Betrand with differentiated products

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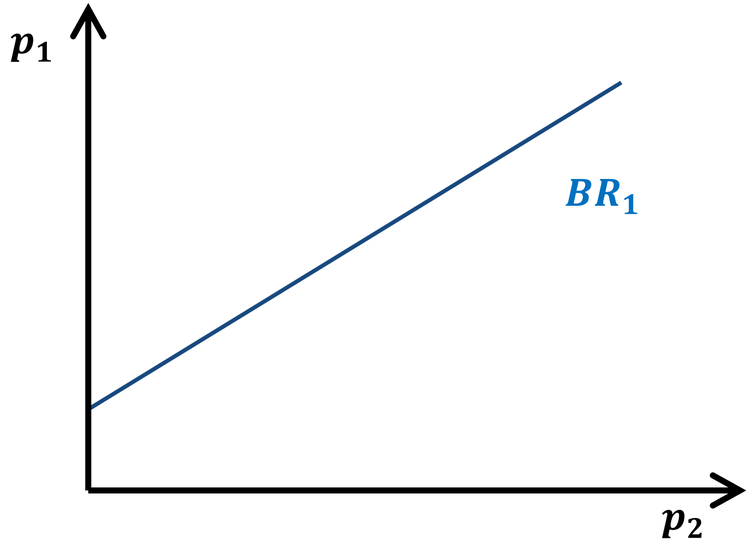

Betrand with differentiated products

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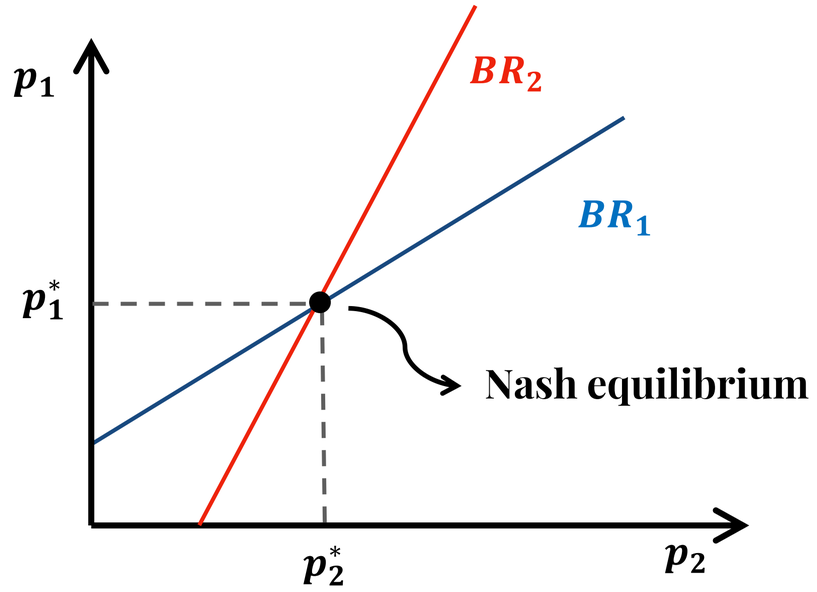
Goal of all firms → Maximize profits

$$\max_{p_2} \pi_2 = (P_2 - c)q_2$$

$$\frac{d\pi}{dp_2} = 0 \leftrightarrow (\dots) \leftrightarrow p_2 = \frac{a + cb}{2b} + \frac{d}{2b}p_1$$



Bertrand with differentiated products

EXERCISE

9. Two firms sell imperfectly differentiated products, denoted 1 and 2, whose demand functions are $q_1 = 10 - p_1 + p_2$ and $q_2 = 10 - p_2 + p_1$, respectively. Each produces its product at a constant marginal and average cost of 6, i.e., $c_1 = 6 = c_2$. They compete in prices, which they set simultaneously and independently.

(a) What price will each firm set? How much will each sell? What profit will each attain? Quantify.

Firm 1 has embarked on an R&D project that has lowered its constant marginal and average cost to 2.

(b) What price will each firm set? How much will each sell? What profit will each attain? Quantify.

Bertrand with differentiated products

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Suppose that firm 2 is unaware of firm 1's R&D project. This gives rise to the **direct effect**. Suppose now that firm 2 becomes aware of the R&D project. This would give rise to additional price changes, which constitute the **strategic effect**. The two together yield the **total effect** of the R&D project.

(c) What is the **direct effect** of the R&D project on firm 1's decision variable, i.e., its **price**? And the **strategic effect**? Quantify and explain.

(d) What is the **direct effect** of the R&D project on firm 1's **profit**? And the **strategic effect**? Quantify and explain.

Recommended readings

CABRAL, LUIS MB. INTRODUCTION TO INDUSTRIAL ORGANIZATION. MIT PRESS, 2017.

- ✓ Chapter 7.1: The Bertrand Model