Exercises for Industrial Organization

Several IO Teams^{*} Instructor: Prof. Vasco Santos

February 18, 2025

^{*}We feel the need to omit any specific name as the author in this notebook, mainly because this compilation is the work not only of the current TAs, but also of many past generations of IO TAs, and given that more than one would remain anonymous, it has been decided to omit us all under the name of IO Team.

Introduction

In this notebook, we provide exercises that will prove useful to deepen the understanding of the topics of Industrial Organization, and therefore, to solve the midterm and exam of the course.

The practical classes will consist in solving some of these exercises. We strongly advise that you read the exercises that are going to be solved in these classes, **before** the lectures. Even further, as detailed solutions are not provided here, individual work along with comparing solutions with your study group is a proven strategy to learn and prepare evaluations.

This notebook does not contain any theoretical material, and therefore it obviously does not replace the use of the official material suggested in the Syllabus [e.g. theoretical lectures, books].

Now, we wish you a nice experience and a good learning.

Contents

1	The Extremes: Perfect Competition and Monopoly	4
2	Market Structure	5
3	Game Theory: Basics	8
4	Oligopoly Models	9
	4.1 Dominant Firm and Monopolistic Competition	9
	4.2 Cournot	10
	4.3 Stackelberg	12
	4.4 Bertrand	12
5	Market Power	14
6	Barriers to Entry	16
7	Bibliography	17

1 The Extremes: Perfect Competition and Monopoly

- 1. Consider the industry of Portuguese footwear, which is a perfectly competitive one with a demand function given by P = a bQ. In this market every firm sells the same product and they all have the same cost structure, TC = cQ.
 - (a) Define the long-run equilibrium price, quantity (aggregate and firm level) and profit.
 - (b) Draw and calculate the consumer surplus, the producer surplus and the total welfare.
- 2. Assume that the demand of a given perfectly competitive industry is given by P = 40 Q. Furthermore, consider that in this industry there are 10 companies, all presenting the same cost function $TC = 5 + 4q + q^2$.
 - (a) Compute the firm-level supply function.
 - (b) Derive the industry-level supply function.
 - (c) Calculate the price and aggregated quantity of equilibrium, as well as firm-level quantity.
 - (d) What is the profit of each firm?
 - (e) Is this a short-run equilibrium or a long-run one? Explain. What do you expect to happen in this market?
- 3. Consider a market supplied by a single firm, The Monopolist. This company faces a market demand of P = a bQ and it has a total cost TC = cQ.
 - (a) Formalize the problem of the Monopolist, and find the equilibrium quantity and price.
 - (b) Draw and calculate the consumer surplus, the producer surplus and the total welfare.
 - (c) Compare the results of the Perfect Competition exercise (first one in this section) and point out the effect over:
 - i. Price
 - ii. Total quantity
 - iii. Firm profits
 - iv. Consumer Surplus
 - v. Producer Surplus
 - vi. Total welfare
 - (d) Draw and calculate the Deadweight loss (excess burden) created by this monopoly.
- 4. Grace Inc. is the only firm in that operates in the market of product Z, a valuable good that is produced in Burkina Faso. This monopolist has a total cost function of $TC = q^2 + 12$. Additionally it faces a demand curve that is given by P = 24 q.
 - (a) Calculate the marginal revenue for Grace Inc.
 - (b) Compute the equilibrium price and quantity of the monopolist.
 - (c) Determine the profit of Grace Inc. in equilibrium.
 - (d) Calculate the elasticity of demand at the equilibrium. What would happen to total revenue if Grace Inc. decreased its price? Is it advisable to do so?
- 5. Show that the optimal price in a Monopoly satisfies the following condition:

$$\frac{P-C'}{P}=-\frac{1}{\epsilon}$$

where C' is the marginal cost and ϵ is the elasticity of demand. When does this solution converge to the perfect competitive outcome? Explain.

Spring 2025

2 Market Structure

1. With table 1, find the interval of values for the Herfindahl Index for the PC market in the US. Find the interval of values for the instability index.

Firm	Marke	et Share
F 11 111	2020	2024
HP	30%	24,2%
Dell	25%	22,3%
Lenovo	15%	17,2%
Apple	8%	15,8%
Others	22%	20,5%

Table 1: Exercise 1.

- 2. (*) Alexis Jacquemin presented the following characteristics for a good measure of concentration:
 - **Non-ambiguous character** Given two markets, it should be possible to say without any doubt which one is more concentrated.
 - Scale invariance The measure should depend only upon the relative dimension of each firm.
 - **Transference** The measure should increase when we reduce the market share of a small firm in favor of a bigger firm.
 - Monotonicity in the number of firms If the N firms have identical market shares, then the measure should be decreasing in N.
 - **Cardinality** Dividing each firm in k equal firms, the measure should decrease in the same proportion.

Verify if the Concentration Ratio (C_k) and the Herfindahl-Hirschman Index (HHI) satisfy these conditions.

3. (*) Show that $H = \frac{1}{N} + N\sigma_i$, where *H* is the Herfindahl index, *N* the number of firms and σ_i the variance of the market shares. With this equation explain the meaning of the measure "equivalent number" of Adelman defined as $NE \equiv \frac{1}{H}$.

4. Table 2 presents the market shares of the 20 biggest insurance firms operating in the life branch. Calculate the possible variation interval for the Herfindahl index in both years. Calculate the variation interval for the Instability Index. What is the equivalent number of Adelman?

Firms	1995	1996
Tranquilidade	16,50%	16,70%
Fidelidade	11,60%	$11,\!60\%$
Ocidental	13,10%	$11,\!60\%$
BPI	12,10%	11,20%
Mundial Confiança	6,10%	$7,\!40\%$
Barclays	2,50%	$6,\!25\%$
Império	5,40%	$5,\!85\%$
BPA	4,90%	4,70%
Aliança UAP	5,00%	4,20%
BFE	2,50%	$2,\!60\%$
Victoria	2,40%	2,00%
Bonança	2,30%	$1,\!90\%$
Europeia	1,80%	$1,\!40\%$
Alico	1,70%	$1,\!40\%$
Abeille	1,40%	$1,\!30\%$
Gan	1,40%	$1,\!30\%$
Eagle Star	1,40%	$1,\!30\%$
Portugal Previdente	1,50%	$1,\!30\%$
Kusitânia	0,40%	$1,\!00\%$
Génesis	1,20%	$0,\!60\%$
Others	4,80%	4,40%
Total	100,00%	$100,\!00\%$

Table 2: Exercise 4.

5. In 2010, the diaper industry in Portugal consisted of 5 firms producing identical diapers. However, in 2020, other firm(s) entered the market, obtaining a market share of 11%. Showing your computations, fill-in the missing items in table 3. Then, according to each concentration measure that you have studied, find out in which year the industry was more concentrated. Explain and calculate Adelman's equivalent number. Compute the volatility index.

Year	Firms					Cor	ncentra	tion Index	
	1	2	3	4	5	Other(s)	C4	Inf H	Max H
2010	40%	15%	15%	15%	15%	0%			
2020	45%	11%	11%	11%	11%	11%			

Table 3: Exercise 5.

6. In 2008, the Portuguese diapers' industry was characterized by the existence of 8 firms producing identical diapers. Let s_i denote the market share of firm i, i = 1, 2, ..., 8. It has recently been observed that the market shares are given by table 4:

Firm	1	2	3	4	5	6	7	8
s_i	60%	10%	5%	5%	5%	5%	5%	5%

Table 4: Ex	ercise 6
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- (a) Compute the measures C_4 and H for this industry.
- (b) Suppose now that firms 2 and 3 merge and become a single firm. Compute the post-merger concentration measures (C_4 and H). Compute the change¹ in concentration resulting from this merger.
- (c) Now suppose that the merger between 2 and 3 does not work out, so treat them separately. Instead, suppose firms 6, 7 and 8 merge (consider that for both this and the next item of this exercise). Compute the post-merger values (C_4 and H).
- (d) Compute the change in concentration resulting from this merger.
- (e) Now suppose that the Portuguese Competition Authority suggests that a merger should not be challenged if the post-merger Herfindahl index and its change due to the merger are such that:
 - H < 0.1, or
 - $0.1 \le H < 0.18$ and $\Delta H < 0.01$, or
 - $H \ge 0.18$ and $\Delta H \le 0.005$

Determine if any of both mergers is likely to be challenged by the Regulator.

7. Consider that the market for coffee in New Zealand is composed by 5 firms that have the market shares shown in table 5:

Kiwi Coffee	NZ Coffee Inc.	CNZ	Kafé Auckland	Wellington Coffees
40%	10%	20%	12%	18%

Table 5: Exercise 7

- (a) Compute the concentration ratios C_4 and H for this market. Now consider that CNZ takes a risky move and decides to acquire NZ Coffee Inc. and Kafé Auckland in order to get closer to its main rival, KiwiCoffee.
- (b) Compute the post-acquisition concentration ratios $(C_4 \text{ and } H)$.
- (c) What reaction do you think the Antitrust Regulatory Body of New Zealand could have to this operation? Explain.

¹ Change' implies computing $\Delta I = I_1 - I_0$, where I_1 is the post-merger index, and I_0 is the original value of the index.

3 Game Theory: Basics

1. Determine the equilibrium using iterative elimination of dominated strategies in the following game and by checking the best response to each player's strategy. Determine the Nash Equilibrium.

	U	D
U	8,8	0,15
D	15,0	2,2

2. Consider a game in which two firms, firm A and firm B, decide simultaneously about the price they want to charge for a product that they both sell. The following payoff matrix displays the outcomes of the different moves they can take:

		В			
		high	low		
۸	high	2,2	15,-3		
л	low	-3,15	10,10		

Compute the Nash Equilibrium of this game.

3. Determine the equilibrium using iterative elimination of dominated strategies in the following game.

	L	C	R
Т	-1,-2	-2,0	$0,\!0$
Μ	-2,0	0,-2	$0,\!0$
В	0,0	0,0	1,1

4. Determine the Nash Equilibrium of the following game:

	L	С	R
Т	$0,\!4$	4,0	$5,\!3$
Μ	4,0	$0,\!4$	5,3
В	3,5	3,5	6,6

5. Firms 1 and 2 are both considering expanding into a small new geography. Firm 1 has been studying this possibility for a while and is ready to go ahead with the expansion now. Firm 2, on the other hand, will only be able to do so in 6 months. The investment needed to enter the new geography is \in 5M. If they both enter, they will have operational profits of \in 2M. If one of the firms enters the new geography alone, it will have an operational profit of \in 10M. How do you expect Firm 1 to act? Draw the game and find its solution.

6. Consider the following sequential game in extensive form.² Find the equilibrium using Backward induction. Represent the game in the normal form and find the Nash Equilibrium.



4 Oligopoly Models

4.1 Dominant Firm and Monopolistic Competition

- 1. Consider a market with demand given by q = 200 2p. In this market operates a dominant firm and a competitive fringe made of small firms. The small firms are price takers offering an aggregate quantity given by S = p - 70 (p > 70), where p is the price established by the dominant firm. The dominant firm satisfies the residual demand. Determine the optimal solution of the dominant firm when its marginal cost is constant and given by c = 70, c = 45 and c = 20. **Hint:** Drawing a plot may be useful to understand the second case.
- 2. Consider a market, with demand given by p = 20 2q, which is constituted by 3 firms (A, B and C). Firm A is more efficient than the other two, operating with a constant marginal cost equal to 9. The total costs of firms B and C are given by the function $TC_i = q_i * (q_i + 11)$, with i = B, C. Find the equilibrium of the market, supposing firm A acts like a dominant firm and firms B and C like a competitive fringe.
- 3. Comment the following statement: "A dominant firm does not care if the number of firms that compose the competitive fringe increases. Only their efficiency matters."
- 4. Suppose that all firms in a market under a Monopolistic Competition environment face an individual inverse demand of $P = 90 + \frac{20}{n} 4q$, then $n \ge 1$. The total cost function of each firm operating in this market is $TC(q) = q^2 + 414, 05$.
 - (a) Assume that in the short-run there are only 4 firms in the market, a firm named "NSBE" and three other similar competitors. Find the optimal quantity and price as well as the profit earned in the short-run by NSBE. What does the sign of the profit level tell you about potential market entry (or exit)?
 - (b) Monopolistic Competition implies zero profits in long-run equilibrium. Use this fact to find the number of firms and NSBE's quantity and price in the long-run. What do you expect will happen in the long-run equilibrium if the fixed cost F = 414, 05 increases? Justify.
 - (c) Compare the short-run and long-run consumer surplus (CS).
 - (d) Taking into account the computed equilibrium quantities in (a) and (b) and given the total cost function, what can you conclude about the productive efficiency of the firms operating in this market?

 $^{^{2}}$ Simultaneous games can also be drawn in extensive form, but this one is not simultaneous.

- (e) Compute the Lerner index $\left(L = \frac{P-MC}{P}\right)$ when $n = n_0$ and show that it is independent of the number of firms n_0 . Explain intuitively the reason why NSBE's market power does not decrease with the number of competitors when $n \to \infty$.
- 5. Take the Portuguese market for breakfast cereals: 160 producers, each with a total cost function of $TC(q_i) = 5q_i + 5$. Each firm faces individual inverse demand of $P = \frac{100}{n\sqrt{q_i}}$ for which it tries to maximize its profits.
 - (a) Compute the short run equilibrium (individual quantities, prices and profits).
 - (b) If there is free entry and exit in the long run, what will be the equilibrium (individual quantities, prices and profits)?
 - (c) Among the firms still in the market, there is the well-known firm Killogg's. How much would it be willing to "pay" to each competitor in order to make them leave the market?
- 6. Consider a market under a monopolistic competition environment with n = 101 firms with identical demand and cost functions:

$$P = 150 - q_i - \frac{1}{50} \sum_{k=1}^{n-1} q_k$$
$$TC(q_i) = \frac{1}{2} q_i^3 - 20q_i^2 + 270q_i$$

with $i \neq k$ and i = 1, ..., n.

- (a) Assume that the number of firms in the market does not change. Find the optimal quantity and price as well as the profit earned in the short-run by each firm.
- (b) Assume now that there is free entry of new firms. What is the long-run equilibrium in this market?

4.2 Cournot

- 1. Consider the market of an homogeneous product with demand given by p = 100 2q. There are two firms, both with constant marginal cost equal to 10.
 - (a) Compute the Cournot-Nash equilibrium.
 - (b) Calculate the efficiency loss as a percentage of the efficiency loss in the monopoly situation.
 - (c) Repeat the exercise assuming now that there are 8 firms instead of 2 firms.
- 2. Consider a duopoly with demand given by $Q = 10 \frac{p}{2}$. The total cost function of each firm is given by C = 10 + q(q+1). Determine the Cournot equilibrium values.
- 3. Repeat the previous exercise assuming that the cost functions are given by: $C_1 = 10 + 2q_1$ and $C_2 = 10 + 1, 5q_2$.
- 4. Consider a duopoly with demand given by Q = 500 50P. The first firm has a constant marginal cost equal to 8. The second firm has a marginal cost equal to 6 and a limited production capacity of 25 units. Calculate the equilibrium values assuming Cournot competition.

- 5. The shoe industry in a country has 8 firms. 5 of them use an old technology producing 0,25 units per work hour. The others use a modern technology with productivity of 0,45 units per work hour. The market demand is given by Q = 500.000 10p and the wage for each work hour is w=500.
 - (a) Find the market's Cournot equilibrium.
 - (b) Find the maximum value at which a firm would be willing to buy the new technology, assuming that the rest of the firms would continue to use the same technology.
- 6. The US has been pressuring Japan to open the automobile market to models made in the US. Consider that the automobile market in Japan presents a demand curve given by Q = 10 p and is served by three Japanese firms with total costs given by TC = q. The US producers are willing to export automobiles to the Japanese market in function of the price existing there, that is, acting like price-takers, following the function S = p, where S represents the number of US cars sold in Japan if the price in that market was p. The Japanese firms act like Cournot competitors.
 - (a) Suppose Japan is forced to open its automobile market to US exports. What is the price, the quantity produced domestically and the quantity imported in equilibrium?
 - (b) Suppose that the US, instead of asking permission for its firms to sell in Japan, establish an objective quantity for the sale of US-produced automobiles in the Japanese market. Suppose that such quantity is equal to 2. What are the new price, the quantity produced domestically and the quantity imported in equilibrium?
 - (c) Compare, explaining the differences, your answers to the previous questions.
 - (d) Which of the two former systems do the US firms prefer? And the Japanese firms? And the Japanese consumers? And the Japanese society?
- 7. One firm imports computers in order to sell them in the domestic market. It can import each computer by paying 10 to the manufacturer. The domestic demand for this brand equals to q = 100 p. Up to now, this firm has been the only importer of the brand into domestic market. However, the manufacturer of the computer is about to issue a second import license. This license, also allowing the winner to import computers at 10 per unit, will be awarded to the firm willing to pay the most for it. All potential importers interested in importing this brand of which there are more than one as well as the firm that owns the first license. If a firm other than the owner of the first license wins the second, they will compete à la Cournot.
 - (a) Who is going to win the second license? Explain.
 - (b) Will the manufacturer, who produces each computer at a cost of 5, be pleased with the outcome of the auction? What would be your advice to him?
- 8. Consider a market with two firms simultaneously deciding on the technology that they wish to adopt. After this decision they will compete à la Cournot. Note that the competitors observe the first decision before they decide on the quantities. The two available technologies are characterized by the following total cost curves: $TC_a = 10q_a + 120$ and $TC_b = 25q_b + 5$. The demand function is given by P = 40 Q. Find the chosen technologies and the equilibrium in the product market.
- 9. Consider two firms, 1 and 2, producing a homogeneous product, that simultaneously decide how much they want to produce. The market demand is given by D(p) = 100 p. The marginal and average cost of production of both firms is constant and equal to 10. Firm 1 maximizes its profit but the manager of the other firm maximizes a weighted sum of the profit and the quantity produced, i.e., firm 2 maximizes $\pi(q_1, q_2) + aq_2$.
 - (a) Represent graphically the best response functions of both firms.
 - (b) Find the equilibrium quantities.
 - (c) Which value of a maximizes the profit of firm 2?

4.3 Stackelberg

- 1. Consider a duopoly in which two firms offer an homogeneous product and compete in q with no cost to produce. There is a leader firm, say L, and a follower firm, say F. Market demand is p = 10 q.
 - (a) Derive firm F's best response and the equilibrium quantities, price and profits.
 - (b) Assume now that both firms choose q simultaneously. Compare your results and explain the differences.
- 2. Consider a market with 4 firms. Two national (A and B) and the other two foreign (C and D). The foreign firms are price takers and their total costs are: $TC_C(q_C) = \frac{3}{2}q_C^2$ and $TC_D(q_D) = \frac{3}{4}q_D^2$. On the other hand, the national firms compete à la Cournot with average and marginal costs given by $c_a = c_b = 1$. In this market the demand is given by Q = 10 p.
 - (a) Find the foreign supply in this market.
 - (b) Determine the equilibrium quantities, price and profits.
 - (c) (*) Assume that each national firm can choose one and only one of the following strategies. Each of these strategies is associated to different costs.

Strategy	Cost
Eliminate national competitors	2
Eliminate foreign competitors	1
Do nothing	0

Each firm will simultaneously choose its strategy. If both choose the strategy "Eliminate the foreign competitors," they will divide equally the costs.

- i. Calculate the eventual Nash Equilibrium in this game.
- ii. How would your answer change if one of the firms should choose first? Comment the results taking in consideration the eventual benefits and costs for the first firm if it decides to eliminate the national competitor.

4.4 Bertrand

- 1. Two firms, 1 and 2, with constant marginal and average costs equal to 10, operate in a market with demand P = 100 Q, while competing à la Bertrand.
 - (a) What is the market equilibrium?

Firm 1 announces a new marketing strategy: "Never knowingly undersold!" To make this strategy credible, firm 1 adopts as its own the price catalog of firm 2, while dropping its own catalog. Firm 2 knows all these decisions.

- (b) What is the market equilibrium after the implementation of this marketing strategy? Explain your answer while describing the economic intuition behind it.
- (c) Suppose that firm 1, though having its competitor's catalog available for consultation, keeps its own, using the former just to prove that its own price is never higher. What is the market equilibrium resulting from this other marketing strategy?
- (d) Should firm 1 use the "aggressive" marketing strategy described in b)? Answer qualitatively.
- (e) Is firm 2 worse off as a result of it? Quantify.

- (f) Do consumers benefit? Quantify again.
- 2. Consider a duopoly where firms choose prices simultaneously and independently. The demand functions for firms 1 and 2 are given by: $p_1 = 14 q_1 + p_2$ and $p_2 = 14 q_2 + p_1$. Firms face a constant marginal cost equal to 1.
 - (a) Determine the equilibrium prices, quantities and profits.
 - (b) Compare these values with those obtained in the Bertrand model, explaining the differences (in terms of equilibrium prices and profits).
- Consider a duopoly with a differentiated good where demand and cost functions are, for firm 1 and 2 respectively: q₁ = 88 4p₁ + 2p₂ and c₁ = 10q₁ for 1, and q₂ = 56 + 2p₁ 4p₂ and c₂ = 8q₂ for 2. Find the reaction function in prices, assuming that each firm maximizes its profits. Determine the equilibrium price, indicating quantities and profits of each firm.
- 4. In the former exercise, suppose firms choose simultaneously one of the two available technologies before competing in prices. The available technologies have the following total cost functions: $ct_a = 10q_a + 120$ and $ct_b = 25q_b + 5$. Find the chosen technologies and the equilibrium in the goods market.
- 5. (*) Consider the following two period model. In the first period only one firm is present in the market. This monopolist produces 5 units, sold at price P = 32. In the second period, a rival firm enters the market with a slightly differentiated product. The two firms will compete in prices, with both having marginal cost equal to zero. The demands for each firm are given by: $q_1 = a bp_1 + cp_2$ and $q_2 = a bp_2 + cp_1$.
 - (a) Find the equilibrium prices and profits in the second period for a = 90, b = 2 and c = 1.
 - (b) Admit that the firm present in the market in the first period has the possibility to implement the most-favored-costumer clause, i.e., has the possibility to guarantee to each customer in the next period that it would return any difference if the price was smaller in the next period. If this clause is introduced, what will be the equilibrium in the second period? Comment.
- 6. Two firms compete through prices in a market with negatively sloped demand and both have constant marginal costs. One of them (A) starts a project of R&D, with common knowledge cost, which implies a decrease in it's marginal cost. The other firm (B) does not have access to the project.
 - (a) Will consumers always win with this procedure?
 - (b) Will they never win?
 - (c) Will firm B always lose as a consequence of firm A executing the project?
 - (d) Suppose that after the conclusion of the R&D project, firm B has access to it. Would firm B execute it?
 - (e) Will this decision be always socially desirable?
- 7. Comment the following statement: "Oranges and apples are substitutes. Yet, the decision variables of firms who produce them may be strategic complements."
- 8. Two firms, an old one, denoted O, and a new one, denoted N, compete in prices, which they set simultaneously and independently while serving the market for an homogeneous product whose demand equals Q = 10 P.

The old firm's constant marginal and average cost, 2, is common knowledge.

However, the new firm's constant marginal and average cost is only known to itself. On the other

hand, the old firm knows that firm N produces at a constant marginal and average cost, which can equal either 3 or 1. Firm O attaches equal probability, 0.5, to firm N's two possible marginal and average costs. This incomplete **but accurate** information held by the old firm is common knowledge.

- (a) What price should the old firm choose? Quantify and explain.
- (b) What price should the new firm choose? Quantify and explain.
- (c) What is the old firm's expected profit? Quantify and explain.
- (d) What are the new firm's possible profits? Quantify and explain.
- (e) Would it be socially better if firm N's constant marginal and average cost was common knowledge? Quantify and explain.
- 9. Two firms sell imperfectly differentiated products, denoted 1 and 2, whose demand functions are $q_1 = 10 p_1 + p_2$ and $q_2 = 10 p_2 + p_1$, respectively. Each produces its product at a constant marginal and average cost of 6, i.e., $c_1 = 6 = c_2$. They compete in prices, which they set simultaneously and independently.
 - (a) What price will each firm set? How much will each sell? What profit will each attain? Quantify.

Firm 1 has embarked on an R&D project that has lowered its constant marginal and average cost to 2.

(b) What price will each firm set? How much will each sell? What profit will each attain? Quantify.

Suppose that firm 2 is unaware of firm 1's R&D project. This gives rise to the direct effect. Suppose now that firm 2 becomes aware of the R&D project. This would give rise to additional price changes, which constitute the strategic effect. The two together yield the total effect of the R&D project.

- (c) What is the direct effect of the R&D project on firm 1's decision variable, i.e., its price? And the strategic effect? Quantify and explain.
- (d) What is the direct effect of the R&D project on firm 1's profit? And the strategic effect? Quantify and explain.

5 Market Power

- 1. Consider a market with a single demand where two firms compete, defining prices simultaneously. The two firms have been tacitly colluding, practicing a monopoly price and sharing the profits generated by their cooperation equally.
 - (a) What discount factor must these firms have for cooperation to be maintained in the future?

A change in consumers' preferences has made the market more attractive. As such, it is now expected to grow at a rate g per year.

- (b) How does the discount factor required to sustain collusion change?
- (c) Study the impact of g on the discount factor and explain the intuition.

- 2. In the homogeneous wallet market, firm 1 and 2 compete in quantities. The demand curve is P = 100 2Q, where P is price, and Q total demanded quantity. Firm 1 has the following total cost function, $C(q_1) = 20q_1$; while firm 2 has the following cost function, $c(q_2) = 30q_2$.
 - (a) Compute the equilibrium price, quantities and profits for the one-stage game.
 - (b) Let this game allow collusion by becoming an infinitely-repeated game. Which firm will produce as a monopolist now? Why?
 - (c) What is the efficient cartel solution with lateral payments? What is the smallest share of the profits that firm 1 would be willing to accept?
 - (d) For a 75%-25% profit-sharing agreement, which discount factor is needed for it to be sustainable?
 - (e) What is the efficient and equitable solution (i.e., both firms produce the same)? What are the discount factors needed for this kind of cooperation to hold?
- 3. Two firms, A and B, supply a market whose yearly demand is given by q = 10 p. They produce the good traded in this market at a constant marginal and average cost of 4. Firms compete in quantities, which they choose every year, doing so simultaneously and independently, and expect to do so forever.

Suppose first that A and B are not colluding.

(a) How much will each produce per year? What will their yearly profit be?

Suppose now that A and B are tacitly colluding with the aim of maximizing and equally sharing industry profit.

(b) How much will each produce per year? What will their yearly profit be?

Suppose that if either firm deviates from the collusive agreement, the other resorts to playing forever as if they had never colluded.

- (c) What is the condition on the discount factor, δ , that must be obeyed for the two firms to be able to collude?
- (d) What would this condition be if the two firms competed in prices instead of competing in quantities?
- (e) In which case is it easier to sustain a tacit collusion agreement? Intuitively explain why.
- 4. Consider two firms interacting in two identical and independent markets. The markets differ in that in market 1 a firm's price at time t is observed at t + 1, whereas in market 2 it is learnt only at t + 2. Thus, although each of the markets meets every period, market 2 has longer information lags.
 - (a) Derive the set of discount factors such that, in the absence of multimarket contact, collusion in market 2 would be sustainable.
 - (b) Compute the minimum threshold value for the discount factor such that, under multimarket contact, collusion in both markets is sustainable. Hint: if firms can use information from one market to detect (and punish) deviation in *all* markets, in which period would it be better to deviate in market 2? And in market 1?
 - (c) Compare and interpret the results obtained in the two previous items.
- 5. Consider a market with demand given by Q = 40 P. There are two firms, with constant marginal costs $c_1 = 10$ and $c_2 = 11$. Compute the Cournot solution and the following Cartel solutions:

- Efficient Cartel with lateral payments
- Efficient Cartel with half and half division of profits.
- Efficient with equal market shares.
- 6. In market A there are two firms operating. Firm 1 has a 70% market share and firm 2 has a 30% market share. They have been competing à la Bertrand and therefore have no profits in equilibrium. The are considering entering into a collusive agreement, in which firm 1 gets 70% of the monopoly profits and firm 2 gets 30% of the monopoly profits.
 - (a) Which discount factors would make this agreement stable? How are these related with each firm's market share?
 - (b) Generalize for market shares $s_1 = 1 s_2$. How does the asymmetry in the size of the firms impact the stability of collusive agreements?

6 Barriers to Entry

1. Firm 1 is the first firm in a given market. Firm 1 can choose one of two technologies, A and B, with respectively the following cost functions:

$$C_A = 60 + 2q_1$$
 $C_B = 10 + 8q_1$

The inverse demand curve is given by P = 20 - Q where Q is the total output of the industry.

- (a) Which technology would firm 1 choose if its monopoly lasts forever?
- (b) Suppose that firm 2 is considering the possibility of entry in this market and that it can also adopt any of the aforementioned technologies. If firm 2 enters, firms will compete à la Cournot. Knowing this, which technology should firm 1 choose? In this model, what is the welfare effect of the existence of a potential competitor?
- 2. One firm with constant marginal and average costs equal to 10 is in a market with demand P = 100 Q. Another firm is considering to enter in this market with a technology with constant marginal and average cost equal to 30. The new firm can choose between building a plant with capacity 10 or 100. Assume that firms compete ex post à la Cournot and the possibility that the installed (i.e., incumbent) firm acts aggressively.
 - (a) What capacity should the new firm choose? Why?
 - (b) If the firm that is considering entering has access to a technology equal to the technology of the firm already installed, how do your previous answers change?
- 3. Imagine that the demand of some homogeneous product is given by P = 100 2Q. The total cost is given by TC = 10Q. Consider a non-refundable cost of entering the market of S = 100. Nowadays the market is covered by only one firm, but there is some potential competitor.
 - (a) How much will the first firm produce if it remains a monopolist (without potential competitor)?
 - (b) Assuming that the potential competitor enters competing in a Stackelberg fashion, what are the profits for this competitor?
 - (c) If the original firm would like to keep the potential competitor out, how much would she need to produce? What would be the resulting price?

- (d) Assuming that the first firm takes a limit price strategy, compute the Lerner index as a function of S. Explain your result.
- (e) What is the value of S such that, for values below it, the first firm would prefer to avoid a limit price strategy?

7 Bibliography

Several of the exercises you can find here are from the following textbooks. You may use these texts to find even more exercises that are not contained in this document. Furthermore, the vast database of past Midterms and Exams will help you prepare for the assessments.

- Cabral, L. "Economia Industrial". McGraw Hill (1994).
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