Industrial Organization

Midterm Fall 2024 – Solution Topics

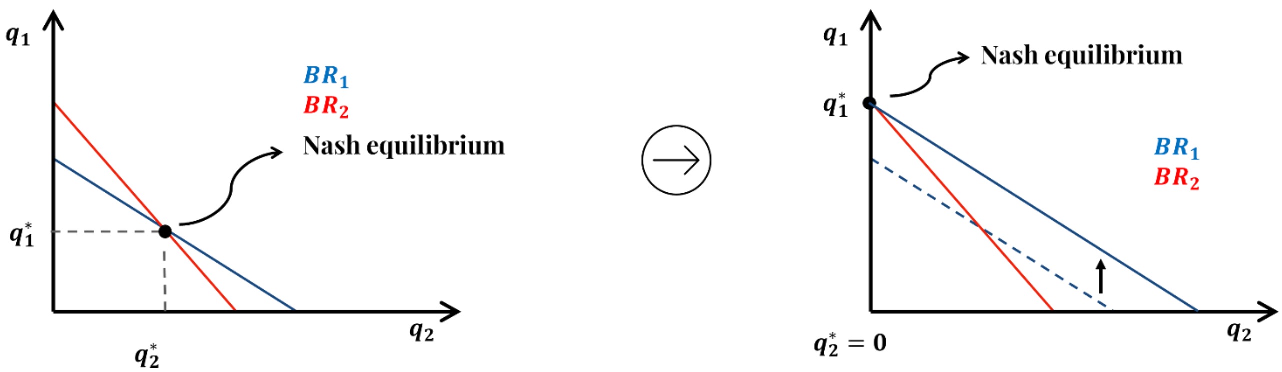
1. False.

If the number of firms in the competitive fringe increases, the aggregate supply of the fringe, $F(P)$, also increases. This expansion in $F(P)$ reduces the residual demand faced by the dominant firm, $D'(P) = D(P) - F(P)$, ultimately altering the market equilibrium.

2. True.

If one of the firms becomes more efficient, its best response function shifts outward.

In this context, the efficiency gain could reduce the firm's marginal cost to a level where the Nash-Cournot equilibrium, determined by the intersection of the firms' best response functions, leads the less efficient firm to produce zero units of output. As a result, the market effectively becomes a monopoly, with the more efficient firm supplying the entire market. In this scenario, the market quantity would equal the monopoly quantity, and the price would rise to the monopoly level.



3.

(i)

Monopoly Model.

Firm 1's profit-maximization problem:

$$\max_{q_1} \pi_1 = P(q_1)q_1 - 4q_1$$

$$FOC: \frac{d\pi_1}{dq_1} = 0 \Leftrightarrow 10 - 2q_1 - 4 = 0 \Leftrightarrow \boldsymbol{q_1^* = 3 \wedge P = 7 \wedge \pi_1 = \pi_2 = (7-4) \times 3 = 9}$$

(ii)

Cournot Model.

Firm 1's profit-maximization problem:

$$\max_{q_1} \pi_1 = P(q_1 + q_2)q_1 - 4q_1$$

$$FOC: \frac{d\pi_1}{dq_1} = 0 \Leftrightarrow 10 - 2q_1 - q_2 - 4 = 0 \Leftrightarrow \boldsymbol{q_1^* = 3 - \frac{q_2}{2}}$$

Firms 1 and 2 face the same demand and have identical cost structures (in particular, equal MC). Hence, $\boldsymbol{q_2^* = 3 - \frac{q_1}{2}}$ and thus, in the Cournot-Nash equilibrium, $\boldsymbol{q_1^* = q_2^*}$.

In equilibrium:

$$\begin{cases} q_1^* = 3 - \frac{q_2}{2} \\ q_1^* = q_2^* \end{cases} \leftrightarrow \begin{cases} q_1^* = 3 - \frac{q_1}{2} \\ - \end{cases} \leftrightarrow \begin{cases} \frac{3}{2} q_1^* = 3 \\ \boldsymbol{q_1^* = q_2^* = 2} \end{cases}$$

$$P = 10 - (2 + 2) = 6$$

$$\boldsymbol{\pi_1 = \pi_2 = (6 - 4) \times 2 = 4}$$

(iii)

Now $MC_1 = MC_2 = c$

Cournot Model.

Firm 1's profit-maximization problem:

$$\max_{q_1} \pi_1 = P(q_1 + q_2)q_1 - cq_1$$

$$FOC: \frac{d\pi_1}{dq_1} = 0 \Leftrightarrow 10 - 2q_1 - q_2 - c = 0 \Leftrightarrow \boldsymbol{q_1^* = \frac{10 - c}{2} - \frac{q_2}{2}}$$

Firms 1 and 2 face the same demand and have identical cost structures (in particular, equal MC). Hence, $\boldsymbol{q_2^* = \frac{10-c}{2} - \frac{q_1}{2}}$ and thus, in the Cournot-Nash equilibrium, $\boldsymbol{q_1^* = q_2^*}$.

(iv)

In equilibrium:

$$\begin{cases} q_1^* = \frac{10 - c}{2} - \frac{q_2}{2} \\ q_1^* = q_2^* \end{cases} \leftrightarrow \begin{cases} q_1^* = \frac{10 - c}{4} - \frac{q_1}{4} \\ - \end{cases} \leftrightarrow \begin{cases} \frac{3}{4} q_1^* = \frac{10 - c}{4} \\ \boldsymbol{q_1^* = q_2^* = \frac{10 - c}{3}} \end{cases}$$

(v)

Firm U's profit-maximization problem:

$$\max_{c} \pi_U = (c - 4)(q_1 + q_2) = (c - 4)\left(\frac{20 - 2c}{3}\right)$$

$$FOC: \frac{d\pi_U}{dc} = 0 \Leftrightarrow \frac{20}{3} - \frac{4c}{3} + \frac{8}{3} = 0 \Leftrightarrow c = 7$$

(vi)

Without the centralized production arrangement - in (ii):

$$P = 6 \wedge q = 4 \wedge CS = \frac{4 \times 4}{2} = 8 \wedge PS = 4 + 4 = 8 \wedge TW = 16$$

With the centralized production arrangement:

$$c = 7 \rightarrow q = \frac{20 - 2 \times 7}{3} = 2 \wedge P = 8$$

$$CS = \frac{2 \times 2}{2} = 2$$

$$PS = 1 + 1 + 6 = 8$$

$$TW = CS + PS = 2 + 8 = 10$$

4.

(i)

In this market, two symmetric firms compete à la Bertrand. As a result, the equilibrium is the well-known Bertrand Paradox where $P_A = P_B = MC_A = MC_B = 2 \wedge \pi_A = \pi_B = 0$.

(ii)

If the industrial accident was only marginal, firms A and B would continue to compete à la Bertrand and have the same marginal costs. This would result in the Bertrand Paradox described in (i), where firm B earns zero profits.

However, if the industrial accident was catastrophic, forcing firm A to exit the market, firm B would become a monopolist and earn positive profits. In this scenario, firm B would charge:

$$\max_{P} \pi_B = (P - 2)(10 - P)$$

$$FOC: \frac{d\pi_B}{dP} = 0 \Leftrightarrow 10 - 2P + 2 = 0 \Leftrightarrow \boldsymbol{P = 6 \wedge q = 4 \wedge \pi_B = 16}$$

As firm B is unaware of the type of accident that actually occurred, it should set its price at the monopoly level. This strategy ensures positive profits if firm A has exited the market due to a catastrophic accident. On the other hand, even if the accident was marginal and firm A remains in the market, firm B's profit from charging the monopoly price would still be zero—equivalent to the outcome of pricing at marginal cost.

Therefore, the expected profit of firm B is: $\boldsymbol{E(\pi_B) = {}^1\!/_2 \times 16 + {}^1\!/_2 \times 0 = 8}$.

(iii)

If the industrial accident was marginal, firm A would remain in the market. Knowing that firm B is charging a price of 6, firm A would undercut by setting a price of $\boldsymbol{P = 6 - \varepsilon}$. By doing so, firm A would capture the entire market, selling $q = 10 - 6 = 4$ units of output. This strategy would yield profits for firm A of $\boldsymbol{\pi_A = (6 - 2) \times 4 = 16}$.

(iv)

In the absence of the accident, both firms would compete in a Bertrand framework, resulting in a price of 2, as outlined in (i).

If the industrial accident were catastrophic, firm B would become a monopolist and set a price of 6, significantly harming consumers due to the price increase.

Conversely, if the accident were marginal, firm A would remain in the market but would undercut firm B by setting a price of $P = 6 - \varepsilon$, as detailed in (iii).

In both scenarios, consumers would be harmed, as the market equilibrium price would rise from 2 to either 6 or $P = 6 - \varepsilon$.

(v)

In the absence of the accident, both firms would compete à la Bertrand, resulting in zero profits for both, as described in (i).

If the industrial accident were catastrophic, firm B would become a monopolist, setting a price of 6. In this case, firm B would earn a profit of 16, as calculated in (ii), while firm A would exit the market, earning zero profits—equivalent to its outcome in the Bertrand competition scenario.

If the industrial accident were only marginal, firm A would remain in the market and undercut firm B, earning a profit of 16, as explained in (iii). Meanwhile, firm B would earn zero profits, the same as in the Bertrand competition scenario.

Therefore, neither firm is negatively impacted by the accident. If the accident is catastrophic, firm B benefits while firm A remains indifferent. If the accident is marginal, firm A benefits while firm B remains indifferent.

(vi)

In the absence of the accident, both firms would compete in a Bertrand framework, leading to a price of 2. In this scenario, each firm's profit would be zero, while consumer surplus would amount to $\frac{8\times8}{2} = 32$. Therefore, total welfare would be 32.

With the industrial accident, the price would rise to either 6 (if the accident were catastrophic) or $6 - \varepsilon$ (if the accident were marginal). As previously calculated, private profits, and hence producer surplus, would be 16, while consumer surplus would decrease to $\frac{4\times4}{2} = 8$. Consequently, total welfare would decline to 24, leaving society worse off with a welfare loss of $\Delta TW = -8$.

(vii)

If firm A does not publicize the type of industrial accident, its profit will depend on the severity of the accident. If the accident was marginal, as previously explained, firm A would earn a profit of 16. However, if the accident was catastrophic, firm A would exit the market and earn zero profit.

If firm A does publicize the type of industrial accident, making firm B aware of its competitive status, the outcomes would differ. If the accident was catastrophic, firm A would leave the market regardless, and thus it would be indifferent to publicizing the type of accident or not. However, if the accident was marginal, firm A would remain in the market, and this information would become common knowledge. In this scenario, firms A and B would compete as usual in a Bertrand framework, resulting in zero profits for both, as detailed in (i). By contrast, withholding this information would allow firm A to profit 16.

Therefore, firm A is better off not publicizing the type of accident.