Advanced Microeconomics

Fall 2024

Final Exam

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1. You have a total of 120 minutes (2 hours) to solve the exam.
2. The use of calculators is not allowed.
3. If you need additional space to answer a question, you can use the back *of the same page*.

Read each question carefully. Good luck!

I (4.5 points)

Consider an economy with two agents, A and B, with preferences over two goods as follows:

$$U_A = x_A^3 . y_A$$
$$U_B = 3x_B + y_B$$

It is known that the initial endowment allocation is $\{(w\}_X^A,\ w_Y^A, w_X^B,\ w_Y^B) = (3,8,7,2)$.

a. (1.5 points) Find the contract curve for this economy.

Setting $\mathrm{MRS}_{x,y}^A = \mathrm{MRS}_{x,y}^B \Leftrightarrow \frac{3x_A^2 \cdot y_A}{x_A^3} = 3 \Leftrightarrow y_A = x_A$

As the total endowments of x and y are (10,10), we can be sure that both origins are a part of the function resulting from the tangency condition, thus ensuring we have the full set of efficient points.

$$\mathrm{C.C.:}\ \mathrm{y_A} = \mathrm{x_A}$$

Grading: 1 point for the tangency condition, 0.5 for the analytical expression

b. (2 points) Find the walrasian equilibrium for this economy.

There were several possible ways to answer this question:

1. The walrasian equilibrium price ratio must be 3 (MRS of Agent 2). Otherwise, Agent 2 would want to consume at a corner, whereas Agent 1 — with Cobb-Douglas preferences — wants to consume positive amounts of both goods, and there would be no equilibrium.
2. From the 1st welfare theorem, the equilibrium must be efficient. Along the contract curve, the MRS of both agents is 3 and that is therefore the only possible equilibrium price (regardless of the initial endowment).
3. Calculate demands for both (including demands for perfect substitutes, that must specify all possible prices and corner solutions) and then find the price ratio that leads to market clearing considering all possible branches in the demands for the agent who sees the goods as perfect substitutes.

Agent 1's preferences:

$$x_A^* = \frac{3}{4} \times \frac{3P_x + 8P_y}{P_x} = \frac{9}{4} + \frac{6P_y}{P_x} \quad y_A^* = \frac{1}{4} \times \frac{3P_x + 8P_y}{P_y} = \frac{3P_x}{4P_y} + 2$$

$$x_A^* = \frac{9}{4} + \frac{6}{3} = \frac{17}{4} \quad y_A^* = \frac{3 \times 3}{4} + 2 = \frac{17}{4}$$

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$$x_B^* = 10 - \frac{17}{4} = \frac{23}{4} \quad y_B^* = 10 - \frac{17}{4} = \frac{23}{4}$$

$$\left(x_A^*, y_A^*, x_B^*, y_B^*, \frac{P_x}{P_y}\right) = \left(\frac{17}{4}, \frac{17}{4}, \frac{23}{4}, \frac{23}{4}, 3\right)$$

Grading: 0.25 for the price ratio, 0.75 for the correct justification of the price ratio, 0.5 for Agent 1's demands and equilibrium allocations, 0.5 for the market clearing conditions and Agent 2's demands.

c. (1 point) Can the allocation $(x_A, y_A, x_B, y_B) = (8,7,2,3)$ become an equilibrium allocation through a redistribution of the initial endowment?

According to the First Welfare Theorem, any competitive equilibrium allocation must be Pareto efficient. Since the given point is not on the contract curve, it is not Pareto efficient. Therefore, it cannot be an equilibrium allocation, as it violates the requirement for efficiency in a competitive equilibrium.

Grading: 0.5 for the First Welfare Theorem, 0.5 for the conclusion

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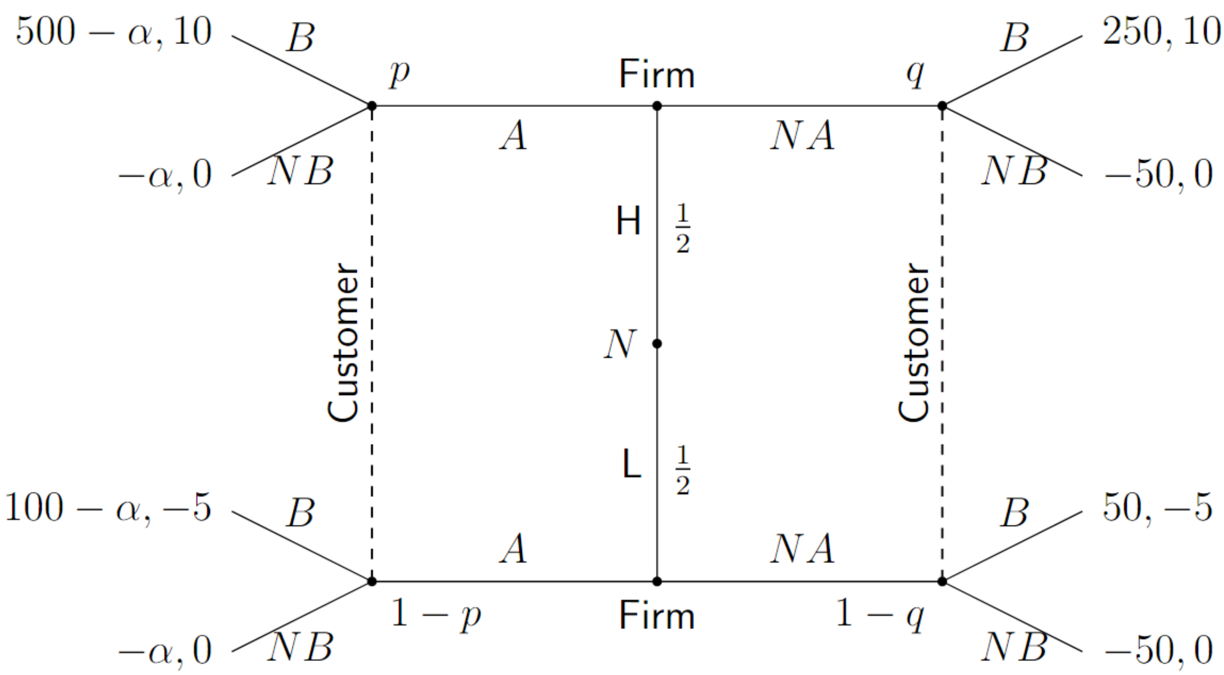
II (5 points)

A firm produces a product which may be High Quality (H) or Low Quality (L). Consumers cannot directly observe the product's quality, but hold a prior belief that it may by High Quality with a probability of 50%. They can, however, observe the level of expenditure on advertising chosen by the firm, which may be either High Advertising (A) or Low Advertising (NA). Consumers will then decide on whether to Buy (B) or Not to Buy (NB) the product.
If the firm sells a High Quality product, the revenue will be 500 with High Advertising, and 300 with Low Advertising; if they sell a Low Quality product, the revenue will be 100 regardless of the level of advertising (as consumers will not repurchase in the future). If the firm does not sell, it gets a revenue of 0.
Furthermore, High Advertising comes with a cost of α, and Low Advertising has a cost of 50.
As for consumers, if they buy a High Quality product, they get a payoff of 10 (already considering the price they paid for it), whereas if they buy a Low Quality product, they get a payoff of -5. If they do not purchase the product, their payoff is 0.

a) (1.5 points) Represent the game in the extensive form.



Grading: 0.9 for the structure, 0.6 for the payoffs

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b) (2 points) Find the values of α for which there is a PBE where firms with High-Quality products spend a lot on advertising, and firms with Low-Quality ones do not. Will there be an informative signal in that case?

If the firm is playing (A,NA), the Customer's beliefs are $p = P(H|A) = \frac{P(A|H)\times P(H)}{P(A)} = 1$ and $q = P(H|NA) = \frac{P(NH|A)\times P(H)}{P(NA)} = 0$.

Given these beliefs, the Customer's payoffs are:

After observing A:

$E_\pi(B) = 10 \times 1 + (-5) \times 0 = 10$

$E_\pi(NB) = 0 \times 1 + 0 \times 0 = 0$

After observing NA:

$E_\pi(B) = 10 \times 0 + (-5) \times 1 = -5$

$E_\pi(NB) = 0 \times 0 + 0 \times 1 = 0$

So the Customer's Best Response is (B,NB)

The Firm's payoffs in response to (B,NB) are:

Type H: $\pi(A) = 500 - \alpha \quad \pi(NA) = -50$

Type L: $\pi(A) = 100 - \alpha \quad \pi(NA) = -50$

The Firm responds with (A,NA) if $(500 - \alpha \geq -50) \wedge (-50 \geq 100 - \alpha) \Leftrightarrow 150 \leq \alpha \leq 550$

Under these conditions, $[(A, NA), (B, NB), p = 1, q = 0]$ constitutes as a separating PBE, and thus the level of advertising functions as an informative signal.

Grading:

0.1 for identifying this as a PBE where firms play (A,NA)

0.2 for each belief p=1 q=0

0.5 for the Customer's best response (B,NB)

0.6 for the conditions that guarantee (A,NA) can be a best response for firms

0.4 for finding the minimum and maximum values of alpha

0.2 for stating the level of advertising constitutes an informative signal

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c) (1.5 points) Let α =100. Is there a PBE where both types choose a high level of expenditure on advertising?

If the firm is playing (A,A), the Customer's beliefs are $p = P(H|A) = \frac{P(A|H) \times P(H)}{P(A)} = \frac{1}{2}$ and q is out of the equilibrium path, so it may take on any value $q \in [0,1]$.

Given these beliefs, the Customer's payoffs are:
After observing A:
$E_{\pi}(B) = 10 \times 0.5 + (-5) \times 0.5 = 2.5$
$E_{\pi}(NB) = 0 \times 0.5 + 0 \times 0.5 = 0$
After observing NA:
$E_{\pi}(B) = 10 \times q + (-5) \times (1-q) = 15q - 5$
$E_{\pi}(NB) = 0 \times q + 0 \times (1-q) = 0$
The Customer's Best Response is (B,B) if $15q - 5 \geq 0 \Leftrightarrow q \geq \frac{1}{3}$, and (B,NB) if $q \leq \frac{1}{3}$.

The Firms Payoffs in response to (B,B):
Type H: $\pi(A) = 400 \quad \pi(NA) = 250$
Type L: $\pi(A) = 0 \quad \pi(NA) = 50$
A Low-Quality Firm would deviate, thus we don't have a PBE if $q \geq \frac{1}{3}$

The Firm's payoffs in response to (B,NB) are:
Type H: $\pi(A) = 400 \quad \pi(NA) = -50$
Type L: $\pi(A) = 0 \quad \pi(NA) = -50$
Neither Firm type would deviate, thus we have a PBE if $q \leq \frac{1}{3}$

PBE: $\left[(A, A), (B, NB), p = 1, q \leq \frac{1}{3}\right]$

Grading:
0.1 for identifying this as a PBE where firms play (A,NA)
0.1 for each belief p=1/2, q is free
0.5 for the Customer's best response (B,B) if q>1/3 and (B,NB) for q<1/3
0.35 for concluding that if Customer's play (B,B), firms will choose (A,NA)
0.35 for concluding that if Customer's play (B,NB) firms will choose (A,NA)

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III (2 points)

A game has two players but player 2 can be of type A (with probability 2/3) or type B (and player 1 does not know player 2's type). The payoff matrices are:

Type A

1\2	L	R
U	(6,0)	(8,1)
D	(4,1)	(2,3)

Type B

1\2	L	R
U	(3,0)	(4,0)
D	(1,1)	(7,5)

Find all the *pure-strategy* Bayes-Nash equilibria.

If 1 plays U, 2’s best response is (R,L) or (R,R).
If 2 plays (R,L), 1’s expected payoff from U is 8*2/3+3*1/3=19/3 and 1´s expected payoff from D is 2*2/3+1*1/3=5/3. 1’s best response to (R,L) will therefore be U.
[U,(RL)] is a BNE.
If 2 plays (R,R), 1’s expected payoff from U is 8*2/3+4*1/3=20/3 and 1´s expected payoff from D is 2*2/3+7*1/3=11/3. 1’s best response to (R,R) will therefore be U.
[U,(RR)] is a BNE.

If 1 plays D, 2’s best response is (R,R). If 2 plays (R,R), 1’s best response is U and we have no BNE.

Grading: 0.75 for each equilibrium, 0.5 for the conclusion of no BNE with D.

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IV (4.5 points)

Anne is a journalist for The Microeconomist, an economics newspaper. She can either exert high effort (e_H) or low effort (e_L), where low effort does not give her any disutility, but high effort gives her a disutility of d_H. Let w denote her income. Anne's utility function is given by $\mathrm{u} = \sqrt{w} - d_e$, where d_e is a disutility term for Anne's effort choice. Anne's reservation utility is 0.
The Microeconomist can either have a good article (G), which gives it a revenue of 400 or a bad article (B), which gives it a revenue of 300. The administration of The Microeconomist only cares about its expected profits and the probability of each outcome depends on Anne's effort, which is given in the following table:

	High Effort	Low Effort
Good Article	¾	½
Bad Article	¼	½

a) (1 point) What is Anne's attitude towards risk (with respect to income)?

Anne is risk averse because the utility function is strictly concave.

Grading: 0.25 for identifying risk aversion, 0.75 for the justification.

b) (1.5 points) Consider the case where effort is observable. If The Microeconomist administration were indifferent between offering Anne a contract where she exerts low effort and offering her a contract where she exerts high effort, what would the value of d_H be?

For the case of high effort, Anne accepts the offer only if $\sqrt{w_H} - d_H \geq 0 \Leftrightarrow w_H \geq {d_H}^2$, so to minimize costs the firm sets $w_H = {d_H}^2$ in which case $\mathrm{E}[\pi|e = e_H] = 375 - {d_H}^2$.

For the case of low effort, Anne accepts the offer only if $\sqrt{w_L} \geq 0 \Leftrightarrow w_L \geq 0$, so to minimize costs the firm sets $w_L = 0$ in which case $\mathrm{E}[\pi|e = e_L] = 350$.

Therefore, for $\mathrm{E}[\pi|e = e_H] = \mathrm{E}[\pi|e = e_L]$, we need ${d_H}^2 = 25$ and $d_H = 5$.

Grading: 0.5 for low effort wage, 0.5 for high effort wage, 0.5 for expected profits and conclusion

c) (2 points) Now let d_H=1. What is the best contract that the Microeconomist can offer Anne, in case effort is not observable?

To make Anne choose e_H over e_L it must be the case that:

$$\frac{3}{4}\left(\sqrt{w_g} - 1\right) + \frac{1}{4}\left(\sqrt{w_b} - 1\right) \geq \frac{1}{2}\sqrt{w_g} + \frac{1}{2}\sqrt{w_b} \Leftrightarrow \frac{1}{4}\sqrt{w_g} - \frac{1}{4}\sqrt{w_b} \geq 1$$

Where minimizing costs implies $w_b = 0$ so that we have $\frac{1}{4}\sqrt{w_g} \geq 1 \Leftrightarrow w_g \geq 16$, which again implies $w_g =$ 16 if costs are minimized.
In this case the expected profits of the firm would be:

$$\mathrm{E}[\pi|e = e_H] = \frac{3}{4}(400 - 16) + \frac{1}{4}(300 - 0) = 363$$

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Alternatively, to make Anne choose e_L over e_H while minimizing costs, it suffices to set $w_g = w_b = 0$, in which case expected profits of the firm will be 350.
The Microeconomist will then choose to offer the contract for high effort, $w_g = 16$ and $w_b = 0$.

Grading: 1 for high effort wages, 0.5 for low effort wages, 0.5 for expected profits and conclusion.

V (4 points)

You want to set up an auction to sell the famous Hope Diamond, part of the Smithsonian collection. You know that some people believe that the diamond is cursed and value it at 16 (million) dollars, whereas others value the diamond at 24 (million) dollars. You estimate that 1/4 of the people believe that it is cursed.

You are aware of two potential buyers for the Hope Diamond. Each buyer's utility will be equal to the difference between their valuation and the price they pay, if they win the auction (and 0 otherwise).

a) (1 point) Do you think a first-price auction would be better than an English auction in this setting?

English Auction is strategically equivalent to the second-price action. Revenue equivalence between first-price and second-price auctions holds (if agents are risk neutral, since they seem to be - and valuations are independent). If agents were risk-averse, the first-price auction would be preferred by seller.

Grading: 0.25 for equivalence between the English and second-price auctions; 0.5 for the revenue equivalence result (including conditions) and 0.25 for right mention of consequences of risk-aversion.

b) (3 points) Find the optimal auction mechanism.

The principal will solve the following problem:

$$\max_{\mathrm{M(24),M(16),P(24,16),P(24,24),P(16,24),P(16,16)}} \frac{3}{4} * M(24) + \frac{1}{4} * M(16)$$

$$\begin{aligned} \text{s.t. } & 24\mathrm{P}(24) - \mathrm{M}(24) \geq 0 \\ & 16\mathrm{P}(16) - \mathrm{M}(16) \geq 0 \\ & 24\mathrm{P}(24) - \mathrm{M}(24) \geq 24\mathrm{P}(16) - \mathrm{M}(16) \\ & 16\mathrm{P}(16) - \mathrm{M}(16) \geq 16\mathrm{P}(24) - \mathrm{M}(24) \end{aligned}$$

IC_{24} and IR_{16} guarantee IR_{24}: $24\mathrm{P}(24) - \mathrm{M}(24) \geq 24\mathrm{P}(16) - \mathrm{M}(16) \geq 16\mathrm{P}(16) - \mathrm{M}(16) \geq 0$; hence $24\mathrm{P}(24) - \mathrm{M}(24) \geq 0$ (IR_{24})

Simplified problem becomes:

$$\max_{\mathrm{M(24),M(16),P(24,16),P(24,24),P(16,24),P(16,16)}} \frac{3}{4} * M(24) + \frac{1}{4} * M(16)$$

$$\begin{aligned} \text{s.t. } & 16\mathrm{P}(16) - \mathrm{M}(16) = 0 \\ & 24\mathrm{P}(24) - \mathrm{M}(24) = 24\mathrm{P}(16) - \mathrm{M}(16) \end{aligned}$$

$16\mathrm{P}(16) - \mathrm{M}(16) = 0 \leftrightarrow M(16) = 16\mathrm{P}(16)$
$\mathrm{M}(24) = 24\mathrm{P}(24) - 8\mathrm{P}(16)$

$$\max \tfrac{3}{4} * M(24) + \tfrac{1}{4} * M(16) = 18\mathrm{P}(24) - 2\mathrm{P}(16) =$$
$$= 18\left(\frac{3}{4} * P(24{,}24) + \frac{1}{4} * P(24{,}16)\right) - 2\left(\frac{1}{4}P(16{,}16) + \frac{3}{4}P(16{,}24)\right)$$
$$= \frac{27}{2}\mathrm{P}(24{,}24) + \frac{9}{2}\mathrm{P}(24{,}16) - \frac{1}{2}\mathrm{P}(16{,}16) - \frac{3}{2}\mathrm{P}(16{,}24) =$$
$$= \frac{27}{2}\mathrm{P}(24{,}24) + 6\mathrm{P}(24{,}16) - \frac{1}{2}\mathrm{P}(16{,}16) - \frac{3}{2}$$

To maximize objective function, set $\mathrm{P}(24{,}24) = \frac{1}{2}$, $\mathrm{P}(24{,}16) = 1$, $\mathrm{P}(16{,}16) = 0$ (and $\mathrm{P}(16{,}24) = 0$), yielding $\mathrm{P}(24) = \frac{5}{8}$, $\mathrm{P}(16) = 0$, $\mathrm{M}(16) = 0$ and $\mathrm{M}(24) = 15$

Grading: Formalize the problem (1 point); Identify the binding constraints (0.5 points); Show that IC_4 and IR_1 guarantee IR_4 (0.5 points); Writing the simplified problem, including the replacement of expected probabilities with the probabilities the principal controls (0.5 points); optimal solution (0.5 points)

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