Advanced Microeconomics

Fall 2022

Final Exam

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1. You have a total of 120 minutes (2 hours) to solve the exam.
2. The use of calculators is not allowed.
3. If you need additional space to answer a question, you can use the back *of the same page*.

Read each question carefully. Good luck!

I (4.5 points)

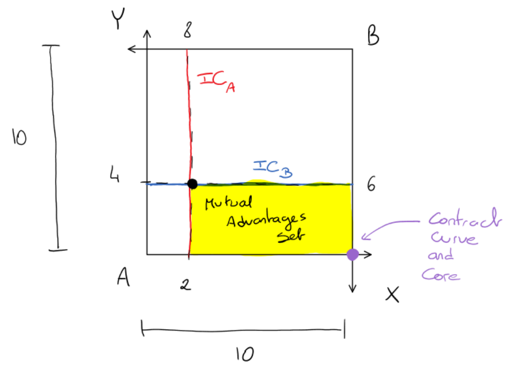
Consider a pure exchange economy with two agents, Alice and Betty, with preferences over two goods given by $u(x_A, y_A) = x_A$ and $u(x_B, y_B) = 4y_B$. It is known that Alice has ω_x^A units of good x and 4 units of good y, while Betty has ω_x^B units of good x and 6 units of good y.

a) (1.5 points) Can the initial endowment be part of a Walrasian Equilibrium? Justify.

No. The initial endowment is not Pareto efficient since we can take some units of y from Alice and give it to Betty without making Alice worse off but while increasing Betty's utility. Moreover, both agents have weakly monotonic preferences so we can apply the first welfare theorem and conclude that, if an allocation is not Pareto Efficient, then it cannot be part of a Walrasian Equilibrium.

Grading:
0.5 Mentioning First Welfare Theorem
0.25 Mentioning the conditions under which the First Welfare Theorem can be applied
0.5 Concluding the initial endowment is not Pareto Efficient
0.25 Concluding the initial endowment cannot be part of a Walrasian Equilibrium

b) (1.5 points) Now, consider that $\omega_x^A = 2$ and $\omega_x^B = 8$. Draw an Edgeworth Box where you identify the core.



Grading:
1.5 For correct graphical representation
-0.75 if only the mutual advantages set is drawn
-0.75 if only the contract curve is drawn
-1.25 if only Anne and Betty's indifference curves at the initial endowment are drawn

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c) (1.5 points) Still considering that $\omega_x^A = 2$ and $\omega_x^B = 8$, find the equilibrium price ratio of this economy and the associated allocation.

Alice's demands are given by $x_A^* = 2 + 4\frac{p_y}{p_x}$ and $y_A^* = 0$

Betty's demands are given by $x_B^* = 0$ and $y_B^* = 6 + 8\frac{p_x}{p_y}$

Markets will clear if $y_A^* + y_B^* = 10 \Leftrightarrow 6 + 8\frac{p_x}{p_y} = 10 \Leftrightarrow \frac{p_x}{p_y} = \frac{1}{2}$

The equilibrium allocation will be: $(x_A, y_A, x_B, y_B) = (10,0,0,10)$

Grading:
0.75 Finding Demands
0.25 Market Clearing Conditions
0.25 Equilibrium Price Ratio
0.25 Equilibrium Allocation

II (4 points)

Amanda wants to buy a new home. There are two types of houses available on the market, undamaged houses (ND) and damaged ones (D). Both are equally likely to show up on the housing market (each has a probability of 0.5). The seller of each type of house can ask an engineer to write a report verifying the house is in good condition (R) or choose not to ask for an engineer's report (NR). Although Amanda cannot observe the condition of house, she can check whether the seller has an engineer's report or not.

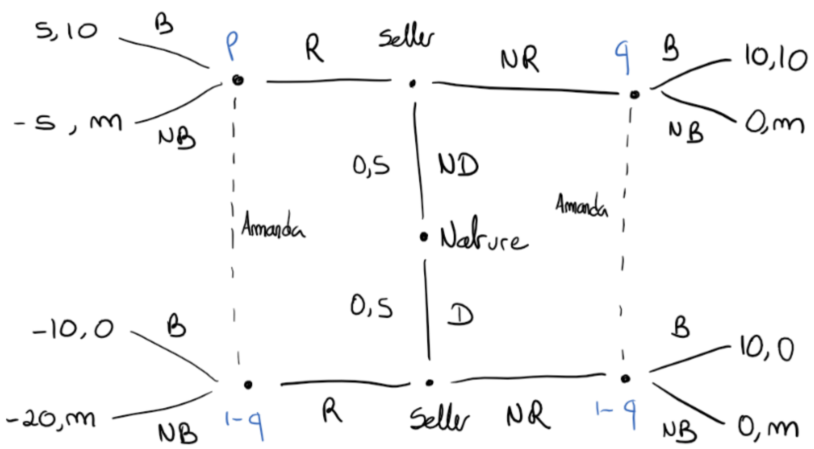
After observing whether the seller has a report or not, Amanda decides whether to buy (B) or not buy (NB) the house.

If she buys the house, her payoff is the difference between her valuation of each type of house and the price she paid for it. Amanda puts a value of 20 on undamaged houses and a value of 10 on damaged houses and all houses are currently sold at a price of 10.

If she chooses not to buy the house, she will remain living with her mother, where she gets a positive payoff of m (where $m > 0$).

The seller's payoff will be the difference between the revenue (the price received for the house if the house is sold; and 0 if the house is not sold) and the cost of the engineer's report. Sellers of undamaged houses only need to pay 5 for an engineer's report, while sellers of damaged houses need to paid a higher price of 20 to get the same report because they need to bribe the engineer to convince her to write it.

a) (1 point) Represent the game in extensive form.



Grading
Structure: 0.6
Payoffs: 0.4

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b) (1.5 points) What's the largest value of m for which there is a Perfect Bayesian equilibrium where only a seller with an undamaged house asks for a report? In this case, can the report constitute an informative signal? Why?

In case the seller is playing (R,NR), the beliefs of Amanda will be $p = P(ND|R) = \frac{P(R|ND)\times P(ND)}{P(R)} = 1$ and $q = P(ND|NR) = \frac{P(NR|ND)\times P(ND)}{P(NR)} = 0$.
Given these beliefs, Amanda's expected payoffs from playing B and NB after R are, respectively, 10 and m, so Amanda chooses B if $m \leq 10$ and NB if $m \geq 10$. Her expected payoffs from playing B and NB after NR are, respectively, 0 and m, so she chooses NB since $m > 0$.

In case Amanda plays (NB,NB), which requires $m \geq 10$, the seller with and undamaged house chooses NR since 0>-5 and the seller of a damaged house chooses NR since 0>-20, so this can't constitute a PBE. In case Amanda plays (B,NB), which requires $m \leq 10$, the seller of an undamaged house chooses R since 5>0 and the seller of a damaged house chooses NR since 0>-10. So we have a PBE here and the maximum value of m that can sustain it is 10.
The certificate produces an informative signal because, under these conditions, it guarantees the existence of a separating PBE.

0.05 Identifying this is a PBE where seller plays R in case the house is undamaged and NR in case the house is damaged
0.2 for beliefs (0.1 for Bayes' rule and 0.1 for final result)
0.4 Amanda's choice after observing R
0.1 Amanda's choice after observing NR
0.2 Seller's best response to (NB,NB)
0.2 Seller's best response to (B,NB)
0.1 concluding a separating PBE exists
0.1 finding that the maximum value of m *that can sustain the equilibrium is 10*
0.15 stating the certificate is an informative signal

c) (1.5 points) Consider that $m = 8$. Is there a Perfect Bayesian equilibrium where neither type of seller asks for an engineer's report?

In case the seller is playing (NR,NR), the beliefs of Amanda will involve p free and $q = P(ND|NR) = \frac{P(NR|ND)\times P(ND)}{P(NR)} = 0.5$.
Given these beliefs, Amanda's expected payoffs from playing B and NB after R are, respectively, $10p$ and 8, so Amanda chooses NB if $p \leq 4/5$ and B if $p \geq 4/5$. Her expected payoffs from playing B and NB after NR are, respectively, 5 and 8, so she chooses NB in this case.

In case Amanda plays (B,NB), which requires $p \geq 4/5$, the seller of an undamaged house chooses R since 5>0 and the seller of a damaged house chooses NR since 0>-10 and we don't have a PBE.

In case Amanda plays (NB,NB), which requires $p \leq 4/5$, the seller with and undamaged house chooses NR since 0>-5 and the seller of a damaged house chooses NR since 0>-20 so that $[(NR,NR),(NB,NB), p \leq 4/5\,, q = 1/2\,]$ is a PBE.

Grading:
0.1 Identifying this is a PBE where seller plays NR in case the house is undamaged and NR in case the house is damaged
0.3 for beliefs (0.1 for Bayes' rule and 0.1 for final result for q and 0.1 for stating p is free)
0.4 Amanda's choice after observing R
0.1 Amanda's choice after observing NR
0.2 Seller's best response to (NB,NB)
0.2 Seller's best response to (B,NB)
0.2 concluding a PBE exists

III (2 points)

A game has two players but player 2 can be of type A (with probability 1/4) or type B (and player 1 does not know player 2's type). The payoff matrices are:

Type A

1\2	L	R
U	(8,0)	(4,2)
D	(1,4)	(2,3)

Type B

1\2	L	R
U	(2,0)	(3,1)
D	(0,2)	(4,4)

Find all the *pure-strategy* Bayes-Nash equilibria.

In case 1 plays U, 2's best response involves choosing R if Type A (to get 2 instead of 0) and R if Type B (to get 1 instead of 0). If 2 plays (R,R), 1's best response is D (given her expected payoff is 3,5 while alternatively choosing U would only yield a payoff of 3.25).
In case 1 plays D, 2's best response involves choosing L if Type A (to get 4 instead of 0) and R if Type B (to get 4 instead of 2). If 2 plays (L,R), 1's best response if U (given her expected payoff is 4.25 alternatively choosing D would only yield a payoff of 3.25).
Therefore, there are no BNE

Grading:
0.4 for 2's best response to U
0.4 for 1's best response to (R,R)
0.1 for concluding no BNE exists here
0.4 for 2's best response to D
0.4 for 1's best response to (L,R)
0.1 for concluding no BNE exists here

IV (5 points)

Paula, a risk-neutral entrepreneur, wants to hire a manager for her online vinyl record store.

Paula has interviewed Dev, whose utility function is $u(w,e) = \sqrt{w} - 4e$ where *w* is wage and *e* is effort level. There are only two possible effort levels, e_H=1 or e_L=0.
Dev is currently earning a wage of 0 at a low-effort job.

There are three possible profit levels for the vinyl record store, and the probability distribution depends on Dev's effort according to the following table:

Profit level	e_H	e_L
64	0.5	0
36	0.5	0.5
0	0	0.5

a. (1.25 points) What would Dev's risk premium be if he was the one facing the lottery composed by these profit levels with the probabilities under e_H? Based on that, what can you say about Dev's attitude towards risk?

Dev's certainty equivalent for the lottery is such that $\sqrt{CE} = \frac{1}{2}(\sqrt{64}) + \frac{1}{2}(\sqrt{36}) \Leftrightarrow \sqrt{CE} = 7$

Since CE=49 and the expected value of the lottery is 50, the risk premium RP=50-49=1.

Given that the risk premium is positive, Dev is risk-averse.

Grading: 0.5 for the CE, 0.25 for the expected value, 0.25 for the risk premium and 0.25 for the conclusion.

b. (1.5 points) What effort level would Paula want to implement if she could observe Dev's effort level?

For the case of high effort, Dev accepts the offer only if $\sqrt{w_H} - 4 \geq 0 \Leftrightarrow w_H \geq 16$, so to minimize costs Paula sets $w_H = 16$ in which case $E[\pi|e = e_H] = 34$.

For the case of low effort, Dev accepts the offer only if $\sqrt{w_L} \geq 0 \Leftrightarrow w_L \geq 0$, so to minimize costs Paula sets $w_L = 0$ in which case $E[\pi|e = e_L] = 18$.

Therefore, given $E[\pi|e = e_H] > E[\pi|e = e_L]$, Paula offers a contract with a wage $w_H = 16$ and where Dev exerts high effort.

Grading: 0.5 for low effort wage, 0.5 for high effort wage, 0.5 for expected profits and conclusion

c. (2.25 points) Assuming that Paula cannot observe Dev's effort level, find the optimal contract that Paula should offer.

To make Dev choose e_H over e_L it must be the case that:

$$\frac{1}{2}\left(\sqrt{w_g} - 4\right) + \frac{1}{2}\left(\sqrt{w_m} - 4\right) \geq \frac{1}{2}\sqrt{w_m} + \frac{1}{2}\sqrt{w_b} \Leftrightarrow \frac{1}{2}\sqrt{w_g} - \frac{1}{2}\sqrt{w_b} \geq 4$$

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Where minimizing costs implies $w_b = 0$ (and $w_m = 0$ since it does not affect Dev's incentives), so that we have $\frac{1}{2}\sqrt{w_g} \geq 4 \Leftrightarrow w_g \geq 64$, which again implies $w_g = 64$ if costs are minimized.
In this case Paula's expected profits would be:

$$E[\pi|e = e_H] = \frac{1}{2}(64 - 64) + \frac{1}{2}(36 - 0) = 18$$

Alternatively, to make Dev choose e_L over e_H while minimizing costs, it suffices to set $w_g = w_m = w_b = 0$, in which case expected profits of the firm will be:

$$E[\pi|e = e_L] = \frac{1}{2}(36 - 0) + \frac{1}{2}(0 - 0) = 18$$

Paula is indifferent between both scenarios and therefore could offer both contracts: one where $w_g = 64$ and $w_m = w_b = 0$; or one where $w_g = w_m = w_b = 0$

Grading: 1 for high effort wages, 0.5 for low effort wages, 0.5 for expected profits, 0.5 for conclusion

V (4.5 points)

Sasha has invested in turning a former convent into a hotel. Sasha knows that there are two potential buyers for the hotel and decides to set up an auction.
You know that with probability 3/4 each buyer will value the hotel at 20 (million) and with probability 1/4 they will value it at 10 (million).
Sasha decided to set up a second price auction with no reservation price.

a) (1 point) Explain whether you think a Dutch auction might perform better than the second-price auction.

Dutch Auction is strategically equivalent to the first -price action. Revenue equivalence between first-price and second-price auctions holds (if agents are risk neutral and valuations are independent). If agents are risk-averse, the Dutch auction is preferred by seller.

Grading: 0.25 for equivalence between the Dutch and first-price auctions; 0.5 for the revenue equivalence result (including conditions) and 0.25 for right mention of consequences of risk-aversion.

b) (1 point) What do you expect buyers to bid in this auction? Why?

In a second-price auction, it is a weakly dominant strategy to bid your true valuation (because the bid is only relevant for the probability of winning the object and not for the payment). Bidding less than the true valuation would lead to the same results as bidding your valuation unless your bid makes you lose the object when you should be winning it.

Grading: 0.5 for the specification of the strategy; 0.5 for justification.

c) (2.5 points) After some additional considerations, Sasha decided to instead set a minimum bid of 20 (million). Assuming Sasha only cares about maximizing the revenue from the sale, what do you think of Sasha's choice?

Sasha solves the following problem:

$$\max_{M(20),M(10),P(20,10),P(20,20),P(10,20),P(10,10)} \frac{3}{4}*M(20)+\frac{1}{4}*M(10)$$

$$\begin{aligned} s.t.\ & 20P(20)-M(20)\geq 0 \\ & 10P(10)-M(10)\geq 0 \\ & 20P(20)-M(20)\geq 20P(10)-M(10) \\ & 10P(10)-M(10)\geq 10P(20)-M(20) \end{aligned}$$

IC_{20} and IR_{10} guarantee IR_{20}: $20P(20)-M(20)\geq 20P(10)-M(10)\geq 10P(10)-M(10)\geq 0$; hence $20P(20)-M(20)\geq 0$ (IR_{30})

Simplified problem becomes:

$$\max_{M(20),M(10),P(20,10),P(20,20),P(10,20),P(10,10)} \frac{3}{4}*M(20)+\frac{1}{4}*M(10)$$

$$\begin{aligned} s.t.\ & 10P(10)-M(10)=0 \\ & 20P(20)-M(20)=20P(10)-M(10) \end{aligned}$$

$$10P(10)-M(10)=0 \leftrightarrow M(10)=10P(10)$$
$$M(20)=20P(20)-10P(10)$$

$$\max \frac{3}{4}*M(20)+\frac{1}{4}*M(10)=15P(20)-5P(10)=$$
$$=15\left(\frac{3}{4}*P(20,20)+\frac{1}{4}*P(20,10)\right)-5\left(\frac{1}{4}P(10,10)+\frac{3}{4}P(10,20)\right)$$
$$=\frac{45}{4}P(20,20)+\frac{15}{4}P(20,10)-\frac{5}{4}P(10,10)-\frac{15}{4}P(10,20)=$$
$$=\frac{45}{4}P(20,20)+\frac{15}{2}P(20,10)-\frac{5}{4}P(10,10)-\frac{15}{4}$$

To maximize objective function, set $P(20,20)=\frac{1}{2}$, $P(20,10)=1$, $P(10,10)=0$ (and $P(10,20)=0$), yielding $P(20)=\frac{5}{8}$, $P(10)=0$, $M(10)=0$ and $M(20)=12.5$

The optimal auction mechanism excludes those with a valuation of 10 and charges the most to those with a valuation of 20. Sasha's choice is correct.

Grading: Formalize the problem (0.75 points); Identify the binding constraints (0.25 points); Show that IC_{20} and IR_{10} guarantee IR_{20} (0.5 points); Writing the simplified problem, including the replacement of expected probabilities with the probabilities the principal controls (0.5 points); optimal solution and conclusion (0.5 points)

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