# Advanced Microeconomics

Spring 2023
Final Exam

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1. You have a total of 120 minutes (2 hours) to solve the exam.
2. The use of calculators is not allowed.
3. If you need additional space to answer a question, you can use the back *of the same page*.

Read each question carefully. Good luck!

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I (5 points)

Consider a pure exchange economy with 2 agents, A and B, with preferences over two goods X and Y. It is known that A's preferences are given by $u_A = \min\{x_A, y_A\}$. B's preferences are weakly monotonic and her demand function for good X is given by $x_B^* = 5\frac{p_y}{p_x} + 5$. Moreover, at the initial endowment, $\omega_x^A = 5$, $\omega_x^B = 10$, $\omega_y^A = 5$ and $\omega_y^B = 10$.

a) (3 points) Find the Walrasian Equilibrium, describing the equilibrium allocation and the equilibrium price ratio. Explain why you do not need to find B's demand for good Y to find the equilibrium.

A's Demands will be given by $x_A^* = 5$ and $y_A^* = 5$ so at the walrasian equilibrium it must be the case that $x_B^* = 10$ and $y_B^* = 10$.
To find the price ratio, at equilibrium we must have that $x_A^* + x_B^* = \omega_x^A + \omega_x^B$, so from here it follows that $5 + 5\frac{p_y}{p_x} + 5 = 15 \Leftrightarrow \frac{p_x}{p_y} = 1$ .
We don't need to find B's demand for good Y to find the equilibrium since we know $y_A^*$ and the equilibrium must verify that $y_B^* = \omega_y^A + \omega_y^B - y_A^*$.

***Grading:***
*Finding A's Demands: 1*
*Obtaining $x_A^*$ and $y_A^*$: 0.5*
*Finding equilibrium quantities (through market clearing or equivalent): 0.5*
*Finding equilibrium price ratio: 0.5*
*Justification for why we don't need demand for $y_B^*$: 0.5*

b) (1 point) Is the initial endowment on the contract curve? Justify.

Yes, because both agents have weakly monotonic preferences the walrasian equilibrium must be pareto efficient by the First Welfare Theorem. As the initial endowment is a walrasian equilibrium it must be Pareto Efficient and is therefore on the contract curve.

Grading:
0.2: For mentioning the initial endowment is a walrasian equilibrium
0.3: For mentioning both agents have weakly monotonic preferences
0.3: For mentioning the first welfare theorem implies walrasian equilibrium is pareto efficient
0.2: For mentioning pareto efficient allocations are points of the contract curve

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c) (1 point) Suppose that agents A and B actually care about each other (and their utility functions would need to change to also incorporate the other agent's utility). Could that change your answer to b)?

If A and B care about each other then we have an externality as A's utility depends on B's consumption levels of the two goods. Therefore, the First Welfare Theorem cannot be applied and so we cannot guarantee the equilibrium (which coincides with the initial endowment) is efficient. Our answer could change.

**Grading:**
*0.4 Mentioning there is an externality problem*
*0.4 Mentioning the First Welfare Theorem cannot be applied*
*0.2 Concluding the answer could change*

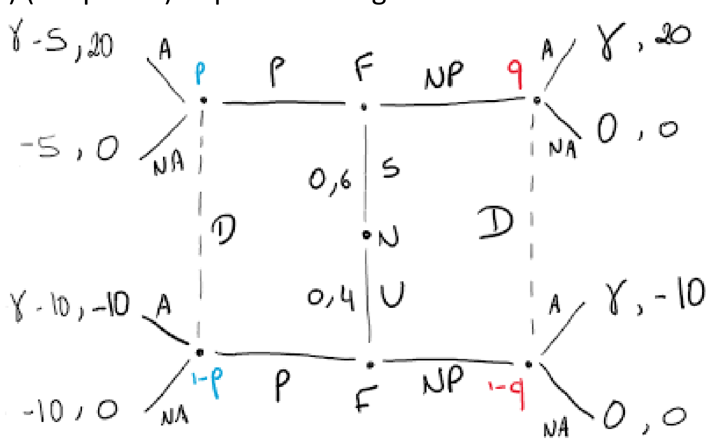
II (5 points)

The MAAT Gala is a famous event where the world's celebrities often show works from the best fashion designers. Dula Pipa (D), a famous singer, is ready to make a statement at the gala. She wants to ask a fashion designer (F) to make her a dress. Fashion designers can either be skilled (S), with 60% probability, or unskilled (U), with 40% probability. Moreover, a designer when making her the offer to work with her can either choose to show Dula a portfolio of their work (P) or not (NP). After checking whether the fashion designer has presented Dula a portfolio or not, she can accept (A) the offer to be dressed by him or her for the Gala or not accept it (NA).

Making a portfolio takes time and effort. Skilled designers face a cost of 5 from showing a portfolio while unskilled designers need to put in longer hours to make a similar portfolio and so face a cost of 10 instead. Dula accepts to be dressed by a fashion designer, the designer will get a benefit of $\gamma$ from having their work shown at the gala while their benefit is 0 if Dula does not accept to be dressed by them. The payoff for a fashion designer is simply the difference between their benefits and costs.

If Dula accepts someone as her fashion designer, she will get a payoff of 20 if the designer is skilled, as she will cause a good impression, but a payoff of -10 if the designer is unskilled. If she does not accept to be dressed by a fashion designer, she gets a payoff of 0.

a) (1.5 points) Represent the game in extensive form.




**Grading:**
*0.9 Structure*
*0.6 Payoffs*

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b) (2 points) Find the minimum and the maximum values of $\gamma$ for which there exists a PBE where skilled designers show a portfolio, but unskilled designers don't. Does the exhibition of the portfolio constitute an informative signal in this case?

If skilled designers show a portfolio and unskilled designer don't, we want to check if there is a PBE where F plays (P,NP). In that case we can compute Dula's beliefs according to Bayes' Rule and obtain that $p = 1$ and $q = 0$. In this case her expected payoff after observing P will be 20 if she chooses A and 0 if she chooses NA so she picks A and her expected payoff after observing NP will be -10 if she picks A and 0 if she picks NA so she will pick NA.

To ensure that we have a PBE, the best response of the fashion designer to Dula must be (P,NP). A skilled designer will choose P if $\gamma - 5 \geq 0 \Leftrightarrow \gamma \geq 5$. An unskilled designer will choose NP if $\gamma - 10 \leq 0 \Leftrightarrow \gamma \leq 10$.

Therefore, the minimum and maximum values of $\gamma$ must be 5 and 10, respectively.
The exhibition of the portfolio indeed constitutes an informative signal because we have a separating PBE.

***Grading:***
*0.1 For identifying this as a PBE where fashion designer picks (P,NP)*
*0.2 For each belief*
*0.5 For Dula's best response*
*0.6 For conditions that guarantee (P,NP) can be the fashion designer's best response*
*0.4 For the minimum and maximum values of $\gamma$*
*0.2 For stating the certificate is an informative signal.*

c) (1.5 points) Show that, regardless of the value of $\gamma$, there always exists a PBE where no designer type will show their portfolio.

If no designer type shows their portfolio then the fashion designer plays (NP,NP) in which case $p$ is free and we can use Bayes' rule to conclude that $q = 0.6$.

Given these beliefs, Dula's expected payoff after observing P will be $30p - 10$ if she chooses A and $0$ if she chooses NA, so she chooses A if $p \geq 1/3$ and NA if $p \leq 1/3$. After observing NP her expected payoff from choosing A will be $8$ and her expected payoff from choosing NA will be 0, so she chooses A.

If she picks (A,A), which requires $p \geq 1/3$, then since $\gamma - 5 < \gamma$ the designer will always pick NP if skilled and the unskilled designer will also pick NP since $\gamma - 10 < \gamma$. In this case we have a PBE that holds regardless of the value of $\gamma$.

***Grading:***
*0.1 For identifying this as a PBE where the fashion designer plays (NP,NP)*
*0.1 For each belief*
*0.6 For Dula's best response (as a function of the belief p)*
*0.35 For concluding that if Dula plays (A,A) a skilled designer picks NP for any $\gamma$*
*0.35 for concluding that if Dula plays (A,A) a skilled designer picks NP for any $\gamma$*

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III (2.5 points)

Serena just inherited 100$. Her best friend, Wilma, suggested that Serena should invest in her firm. For each dollar invested in the firm, there is a chance of 1/3 that the return is 2$ (i.e. Serena gains one additional dollar per dollar invested) and a chance of 2/3 that the return is 0.5$ (i.e. Serena loses half a dollar per dollar invested).
Assume that Serena's utility function is given by $U = w^{1/2}$, where $w$ is wealth.

a) (1.5 points) Show that it is optimal for Serena to invest 0 in the firm.

Serena faces a lottery where, by investing $x$\$ she has a probability of $1/3$ of getting $100 - x + 2x$ (or alternatively $100 + x$) and a probability of $2/3$ of getting $100 - x + 0.5x$ (or $100 - 0.5x$).

Thus, her expected utility will be $E[u] = \frac{1}{3}\sqrt{100 + x} + \frac{2}{3}\sqrt{100 - 0.5x}$

To maximize her expected utility, it must be the case that $\frac{1}{6\sqrt{100+x}} - \frac{1}{6\sqrt{100-0.5x}} = 0 \Leftrightarrow x^* = 0$

***Grading:***
*0.5 to identify the lottery faced by Serena (probabilities and outcomes)*
*0.25 to identify the expected utility of Serena*
*0.25 to maximize the expected utility of Serena*
*0.5 to find the value that maximizes Serena's expected utility*

b) (1 point) Write down the expression that would allow you to find the certainty equivalent for the lottery associated to an investment of $x$ dollars in the firm.

$$u(CE) = E[u] \Leftrightarrow \sqrt{CE} = \frac{1}{3}\sqrt{100 - 2x} + \frac{2}{3}\sqrt{100 - 0.5x} \Leftrightarrow CE = \left(\frac{1}{3}\sqrt{100 + x} + \frac{2}{3}\sqrt{100 - 0.5x}\right)^2$$

***Grading:***
*0.5 For recognizing the utility of the certainty equivalent must yield an expected utility equal to that of the lottery.*
*0.5 For writing the correct expression.*

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IV (4 points)

Deanna manages communications for a car software firm. She must decide on all the fairs and events to prepare for and attend – and she can either exert high effort ($e_H$) or low effort ($e_L$). Low effort does not give her any disutility ($d_L = 0$), but high effort gives her a disutility of $d_H$, where $d_H > 0$. Let w denote Deanna's wages and let $d_e$ denote her disutility from effort ($e$ =L,H). Her utility function is then given by $u = \sqrt{w-2} - d_e$ and she has a reservation utility of 0.

The firm (that only wants to maximize expected net profits) can end up with three different levels of gross profits. The probability of each outcome depends on Deanna's effort and is given in the following table:

| Profits | High Effort | Low Effort |
|---|---|---|
| 102 | 1/2 | 0 |
| 52 | 1/2 | 1/2 |
| 2 | 0 | 1/2 |

a) (1.5 points) If $d_H = 5$ and if effort were observable, which contract would the firm offer?

For the case of high effort, Deanna accepts the offer only if $\sqrt{w_H - 2} - 5 \geq 0 \Leftrightarrow w_H \geq 27$, so to minimize costs the firm sets $w_H = 27$ in which case $E[\pi|e = e_H] = 50$.

For the case of low effort, Deanna accepts the offer only if $\sqrt{w_L - 2} \geq 0 \Leftrightarrow w_L \geq 2$, so to minimize costs the firm sets $w_L = 2$ in which case $E[\pi|e = e_L] = 25$.

Therefore, given $E[\pi|e = e_H] > E[\pi|e = e_L]$, the firm offers a contract with a wage $w_H = 27$ and where Deanna exerts high effort.

**Grading: 0.5 for low effort wage, 0.5 for high effort wage, 0.5 for expected profits and conclusion**

b) (2.5 points) If effort is not observable, find the value of $d_H$ that would now make the firm indifferent between offering a contract that elicits high effort or offering a contract that elicits low effort?

To make Sandra choose $e_H$ over $e_L$ it must be the case that:

$$\frac{1}{2}\left(\sqrt{w_g - 2}\right) + \frac{1}{2}\left(\sqrt{w_m - 2}\right) - d_H \geq \frac{1}{2}\left(\sqrt{w_m - 2}\right) + \frac{1}{2}\left(\sqrt{w_b - 2}\right) \Leftrightarrow \frac{1}{2}\sqrt{w_g - 2} - \frac{1}{2}\sqrt{w_b - 2} \geq d_H$$

Where minimizing costs implies $w_b = 2$ (and $w_m = 2$, since it does not affect incentives), so that we have $\frac{1}{2}\sqrt{w_g - 2} \geq d_H \Leftrightarrow w_g \geq 2 + 4{d_H}^2$, which again implies $w_g = 2 + 4{d_H}^2$ if costs are minimized.
In this case the expected profits of the firm would be:

$$E[\pi|e = e_H] = \frac{1}{2}\left(102 - 2 - 4{d_H}^2\right) + \frac{1}{2}(52 - 2) = 75 - 2{d_H}^2$$

Alternatively, to make Deanna choose $e_L$ over $e_H$ while minimizing costs, it suffices to set $w_g = w_m = w_b = 2$, in which case expected profits of the firm will be 25

To make the firm indifferent between both contracts, it must be the case that $75 - 2{d_H}^2 = 25 \Leftrightarrow d_H = 5$

**Grading: 1 for high effort wages, 0.5 for low effort wages, 0.5 for expected profits, 0.5 for obtaining $d_H$**

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V (3.5 points)

You have developed a new app and want to try to sell it by setting up an auction. You expect there to be two potential buyers for the app, but you are uncertain about their valuations.
You know that with probability 1/3 each buyer will value the app at 9 (million) and with probability 2/3 they will value it at 4 (million). You also know that each potential buyer has an outside option that guarantees them an expected utility of 1 (million).

a) (1 point) Do you think an English auction might be better than a first-price auction?

English Auction is strategically equivalent to the second-price action. Revenue equivalence between first-price and second-price auctions holds if agents are risk neutral and valuations are independent. If agents are risk-averse, the first-price auction is preferred by seller.

*Grading: 0.25 for equivalence between the English and second-price auctions; 0.5 for the revenue equivalence result (including conditions) and 0.25 for right mention of consequences of risk-aversion.*

b) (2.5 points) Assuming buyers are risk-neutral and their valuations are independent, describe the optimal auction mechanism.

The principal will solve the following problem:

$$\max_{M(9),M(4),P(9,9),P(9,4),P(4,9),P(4,4)} \frac{1}{3} * M(9) + \frac{2}{3} * M(4)$$

$$\begin{aligned} s.t.\ & 9P(9) - M(9) \geq 1 \\ & 4P(4) - M(4) \geq 1 \\ & 9P(9) - M(9) \geq 9P(4) - M(4) \\ & 4P(4) - M(4) \geq 4P(9) - M(9) \end{aligned}$$

IC$_9$ and IR$_4$ guarantee IR$_9$: $9P(9) - M(9) \geq 9P(4) - M(4) \geq 4P(4) - M(4) \geq 1$ ; hence $9P(9) - M(9) \geq 1$ (IR$_9$)

Simplified problem becomes:

$$\max_{M(9),M(4),P(9,9),P(9,4),P(4,9),P(4,4)} \frac{1}{3} * M(9) + \frac{2}{3} * M(4)$$

$$\begin{aligned} s.t.\ & 4P(4) - M(4) = 1 \\ & 9P(9) - M(9) = 9P(4) - M(4) \end{aligned}$$

$$4P(4) - M(4) = 1 \leftrightarrow M(4) = 4P(4) - 1$$
$$M(9) = 9P(9) - 5P(4) - 1$$

$$\max \frac{1}{3} * M(9) + \frac{2}{3} * M(4) = 3P(9) + P(4) - 1 =$$
$$= 3\left(\frac{1}{3} * P(9,9) + \frac{2}{3} * P(9,4)\right) + \left(\frac{1}{3}P(4,9) + \frac{2}{3}P(4,4)\right) - 1$$
$$= P(9,9) + 2P(9,4) + \frac{1}{3}P(4,9) + \frac{2}{3}P(4,4) - 1 =$$
$$= P(9,9) + \frac{5}{3}P(9,4) + \frac{2}{3}P(15,15) - \frac{2}{3}$$

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To maximize objective function, set $P(9,9) = \frac{1}{2}$, $P(9,4) = 1$, $P(4,4) = \frac{1}{2}$ (and $P(4,9) = 0$), yielding $P(9) = \frac{5}{6}$, $P(4) = \frac{1}{3}$, $M(4) = \frac{1}{3}$ and $M(9) = \frac{29}{6}$

*Grading: Formalize the problem, including the right reservation utility in IR constraints (1 point); Identify the binding constraints (0.25 points); Show that $IC_9$ and $IR_4$ guarantee $IR_9$ (0.5 points); Writing the simplified problem, including the replacement of expected probabilities with the probabilities the principal controls (0.5 points); optimal solution (0.25 points)*

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