Problem Set #12

The owner of a rare vinyl album would like to sell it online and is considering doing so at an auction website. The seller already knows that there are two potential buyers and that each buyer can be of two possible types. With probability 1/4, a buyer will be the type that values the object at 20. With probability 3/4, a buyer will value the object at 4. The surplus for each type of buyer will be equal to the difference between her valuation and the price she pays for the object, in case she gets it; and zero otherwise.

a. (1 point) The owner is planning a second-price auction but a friend of the owner suggests a first-price auction instead, claiming the revenue would be higher. Do you think that the friend is right?

b. (3 points) You are also a friend of the owner and want to recommend the optimal auction mechanism. Formalize and solve the problem that will allow you to make a recommendation.

c. (0.5 points) Can you think of a real-life mechanism that would lead to the optimal solution for the auctioneer?

- 2. Suppose that TAP Air Portugal is the only airline that offers flights between Lisbon and Barcelona. TAP cares only about expected revenue. Suppose that the possible customers of TAP are divided into two groups of equal size: executives who travel for business reasons and tourists who travel for leisure. Each executive is willing to pay 1000 for the flight but each tourist is only willing to pay 600 for the same flight. The surplus for each type of traveler will be equal to the difference between her valuation and the price she pays for the flight, in case she gets it; and zero otherwise. Assume that each type has a reservation utility of 0.
- a. Which prices will TAP charge if it knows whether the traveler is an executive or a tourist? What is the profit associated with those prices?

Suppose that the company cannot distinguish between an executive and a tourist and there is only one seat left on the plane.

- b. TAP is hesitating between a first-price sealed bid auction and a second-price sealed bid auction for that remaining seat. Without making any calculations, what would your recommendation be? Why?
- c. Assume that TAP can offer a menu of contracts (for instance, a given price may correspond to a certain probability of actually getting the seat).
 - i. Formalize TAP's maximization problem. What type of problem is this?
 - ii. Which constraints will be binding? Solve the optimization problem.