Problem Set #11

1. You have graduated from Nova SBE with brilliant grades and have set up your own consulting business "IDoConsulting". Unfortunately, you are still the only member of your company, and therefore you can only manage one consulting project at a time. Your utility function is $U(t, e) = t - e^2$, where t is the payment that you receive and e is your effort (i.e. the number of days spent on the project). This effort is observable and will be part of the consulting contract (t, e). Your potential clients (firms) will be of two types: Those for whom your advice is extremely useful (Type H) or only very useful (Type L). More precisely, given (t, e), the expected payoff from the consulting project for a firm type i is $\pi_i(t, e) = \alpha_i e - t$, where $\alpha_H = 4$ and $\alpha_L = 2$. Firms' reservation payoff is zero.

   a. Assume that you can observe firms' type. Derive and explain the consulting contracts that maximize your utility and are accepted by the firms.
   b. Now assume that you cannot observe firms' type, but that you have prior belief that both types are equally likely. Find the profit maximizing menu of contracts (t_H, e_H) and (t_L, e_L) that makes firms accept the contract and reveal their type Explain.

2. Consider a Principal-Agent model with the following elements: The government wants a firm to produce a certain amount of a good (i.e. this is a case of "public procurement"). $S(q) = 2\sqrt{q}$ is the government's utility as a function of quantity produced by the firm. The firms has a reservation utility of 0 and production cost function of $C(q, \alpha) = \frac{41}{90} + \alpha q$. Marginal cost, α, which is not observed by the government, can be of two types: $\alpha_L = 1$ or $\alpha_H = 2$, each with probability $\frac{1}{2}$.

   a. What are the incentive compatibility and the participation constraints in this case?
   b. Compute the first best contract.
   c. Compute the second best contract.