Advanced Microeconomics

Fall 2023
Midterm Exam

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1. You have a total of 80 minutes (1 hour and 20 minutes) to solve the exam.
2. The use of calculators is not allowed.
3. If you need additional space to answer a question, you can use the back *of the same page*.

Read each question carefully. Good luck!

I (4.5 points)

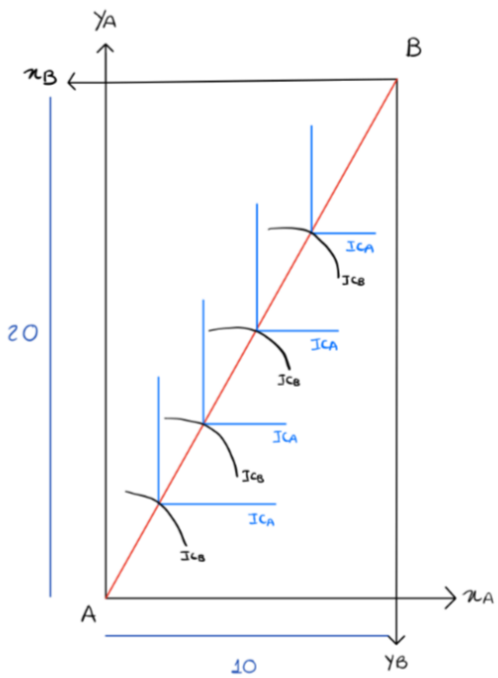
Consider an economy with two agents, A and B, with preferences over two goods as follows:

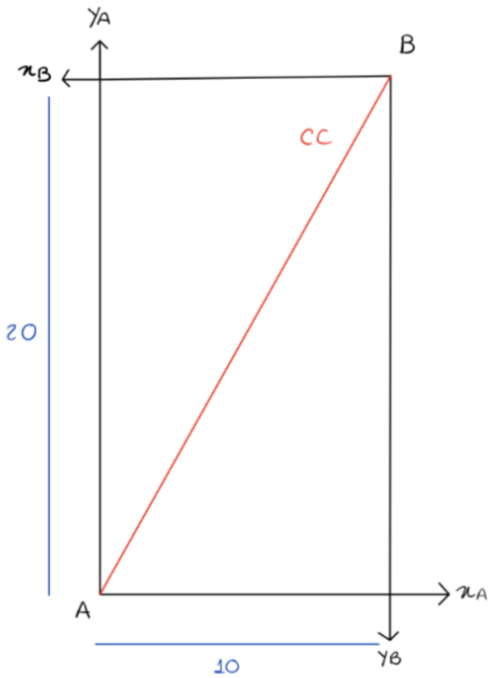
$$U_A = min\{2x_A, y_A\}$$
$$U_B = x_B y_B$$

It is known that, at the initial endowment, $w_X^A + w_X^B = 10$ and $w_Y^A + w_Y^B = 20$.

a) (1.5 points) Draw the contract curve and describe it analytically.

$$CC: y_A = 2x_A$$





Grading:
0.5 points for the justification.
0.5 points for the graphical representation of the contract curve.
0.5 points for the analytical expression.

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b) (1.5 points) Find the utility possibility frontier.

From a), CC: $y_A = 2x_A$

$U_A = \min\{2x_A, 2x_A\} = 2x_A \Rightarrow x_A = \frac{U_A}{2}$

$U_A = \min\{y_A, y_A\} = y_A$

$U_B = x_B y_B \Rightarrow U_B = (10 - x_A)(20 - y_A) \Rightarrow$

$\Leftrightarrow U_B = 200 - 10y_A - 20x_A + x_A y_A \Rightarrow$

$\Rightarrow U_B = 200 - 10U_A - 20 \times \frac{U_A}{2} + \frac{U_A}{2} \times U_A \Rightarrow$

$\Rightarrow U_B = 200 - 20U_A + \frac{U_A^2}{2}$

Grading:

0.5 for using the CC.

0.5 for using the feasibility conditions.

0.5 for writing an expression relating agents' utilities.

c) (1.5 points) Find the **utilitarian** choice for this economy.

(If you haven't replied to c), use the following UPF: $U_B = 200 - 20U_A + \frac{U_A^2}{2}$)

$$\max_{\{U_A, U_B\}} W = U_A + U_B \text{ s.to UPF} \Leftrightarrow \max_{\{U_A\}} W = U_A + 200 - 20U_A + \frac{U_A^2}{2}$$

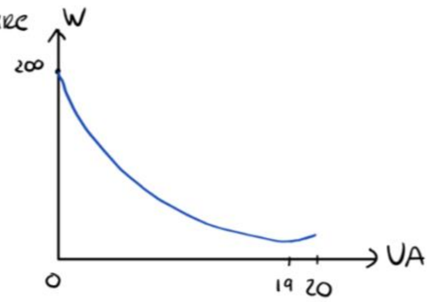
$$\Leftrightarrow \max_{\{U_A\}} W = 200 - 19U_A + \frac{U_A^2}{2}$$

As the function is convex, the maximum(s) is/are located in the extremes of the interval where the function is defined:

$0 \leq U_A \leq 20$

$U_A = 0 \Rightarrow W = 200$

$U_A = 20 \Rightarrow W = 200 - 19 \times 20 + \frac{20^2}{2} = 20$



$U_A = 0 \Rightarrow U_B = 200 - 20 \times 0 + \frac{0^2}{2} = 200$

$U_A = 0 \Rightarrow U_A = \min\{2x_A, y_A\} = 0$

$\min\{2x_A, 2x_A\} = \min\{y_A, y_A\} = 0 \Rightarrow$

$\Rightarrow 2x_A = y_A = 0 \Rightarrow x_A = y_A = 0$

$x_B = 10 - 0 = 10 \quad y_B = 20 - 0 = 20$

$(x_A^*, y_A^*, x_B^*, y_B^*) = (0, 0, 10, 20)$

Grading:
0.5 points for formalization of the problem.
0.5 points for reaching optimal utilities.
0.5 points for reaching optimal allocation.

II (3.5 points)

Consider an economy with two agents, A and B, with preferences over two goods as follows:

$$U_A = x_A + \ln(y_A)$$
$$U_B = 2y_B$$

It is known that, at the initial endowment, $w_X^A = 2$, $w_Y^A = 2$, $w_X^B = 2$, $w_Y^B = 2$.

a) (2.25 points) Find the Walrasian Equilibrium for this economy.

Agent A

$$\max_{\{x_A, y_A\}} U_A = x_A + \ln(y_A)$$
$$\text{s.to } 2p_x + 2p_y = x_A p_x + y_A p_y$$

$$\begin{cases} MRS_{x,y}^A = \frac{p_x}{p_y} \\ 2p_x + 2p_y = x_A p_x + y_A p_y \end{cases} \Leftrightarrow \begin{cases} \frac{1}{\frac{1}{y_A}} = \frac{p_x}{p_y} \\ 2p_x + 2p_y = x_A p_x + y_A p_y \end{cases} \Leftrightarrow \begin{cases} y_A = \frac{p_x}{p_y} \\ 2p_x + 2p_y = x_A p_x + \frac{p_x}{p_y} \times p_y \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \text{---} \\ p_x + 2p_y = x_A p_x \end{cases} \Leftrightarrow \begin{cases} y_A = \frac{p_x}{p_y} \\ x_A = 1 + \frac{2p_y}{p_x} \end{cases}$$

Agent B

$$\max_{\{x_B, y_B\}} U_B = 2y_B$$
$$\text{s.to } 2p_x + 2p_y = x_B p_x + y_B p_y$$

If prices are positive, $x_B = 0$ is optimal.

$$\begin{cases} x_B = 0 \\ 2p_x + 2p_y = x_B p_x + y_B p_y \end{cases} \Leftrightarrow \begin{cases} \text{---} \\ y_B p_y = 2p_x + 2p_y \end{cases} \Leftrightarrow \begin{cases} x_B = 0 \\ y_B = \frac{2p_x + 2p_y}{p_y} = \frac{2p_x}{p_y} + 2 \end{cases}$$

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Equilibrium

$$x_A + x_B = 4 \Rightarrow 1 + \frac{2p_y}{p_x} + 0 = 4 \Rightarrow \frac{p_y}{p_x} = \frac{3}{2} \Rightarrow \frac{p_x}{p_y} = \frac{2}{3}$$

$$y_A + y_B = 4 \Rightarrow \frac{p_x}{p_y} + \frac{2p_x}{p_y} + 2 = 4 \Rightarrow \frac{3p_x}{p_y} = 2 \Rightarrow \frac{p_x}{p_y} = \frac{2}{3}$$

$$x_A^* = 1 + 2 \times \frac{3}{2} = 4 \qquad y_A^* = \frac{2}{3}$$

$$x_B^* = 0 \qquad y_B^* = 2 \times \frac{2}{3} + 2 = \frac{4}{3} + \frac{6}{3} = \frac{10}{3}$$

$$\left(x_A^*, y_A^*, x_B^*, y_B^*, \frac{p_x^*}{p_y}\right) = \left(4, \frac{2}{3}, 0, \frac{10}{3}, \frac{2}{3}\right)$$

Grading:

0.75 points for computing the demand functions of Agent A.

0.75 point for computing the demand functions of Agent B.

0.75 points for computing the equilibrium allocation and price ratio.

b) (1.25 points) Without additional calculations, can you determine if the Walrasian equilibrium allocation will be in the core of this economy?

- The core is the intersection between the (weak) Mutual Advantages Set and the Contract Curve.
- Following the First Welfare Theorem, we conclude that the Walrasian equilibrium allocation is Pareto Efficient. Therefore, the Walrasian equilibrium allocation is part of the Contract Curve, which is the set of Pareto Efficient Points.
- The initial endowment allocation is affordable and is on the budget constraint, hence the equilibrium choice is at least as good as the initial endowment allocation. Hence, the Walrasian equilibrium allocation is also part of the Mutual Advantages Set.
- As the Walrasian equilibrium allocation is simultaneously part of the Contract Curve and of the Mutual Advantages set, it is in the core of this economy.

Grading:

0.4 points for justifying and concluding that the equilibrium allocation is on the Contract Curve.

0.4 points for justifying and concluding that the equilibrium allocation is on the Mutual Advantages Set.

0.45 points for defining the concept of “core” and concluding that the Walrasian equilibrium allocation will be in the core of this economy.

III (4.5 points)

Assume there are three individuals who share an office space. Let F denote the level of smoke in the room. Person 1 and Person 2 both have a marginal cost of 1 associated with smoke. Person 3, however, derives benefits from smoke according to the marginal benefit function $4/\sqrt{F}$.
The price of a unit of smoke is 2 (reflecting its marginal cost expressed in monetary units).

a) (1 point) Assume initially there is the right to smoke. How much would person 3 choose to smoke?

Person 3 solves $4/\sqrt{F}=2$ and chooses F=4.

Grading: 0.5 for the MB=MC condition, 0.5 for the conclusion

b) (1.5 points) Find the Pareto efficient level of smoke.

Samuelson condition: $4/\sqrt{F} - 1 - 1 = 2$ and $F^*=1$

Grading: 1 for the right MB and MC, 0.5 for the conclusion

c) (1 point) Suppose you were a policymaker with the possibility of imposing a tax on agent 3. What tax would you charge to achieve the level you found in b.?

Pigouvian tax, equal to the marginal external cost of 1+1=2.

Grading: 0.25 for mentioning Pigouvian tax, 0.5 for the marginal external cost, 0.25 for conclusion.

d) (1 point) Specify how you would be able to achieve a unanimous choice of the level you found in b. through a Lindahl tax and subsidy policy.

In order to get a unanimous choice of 1, Person 1 would need to be charged a Lindahl tax of 2, Person 2 and Person 3 would each need to receive a Lindahl subsidy of 1.

Grading: 0.75 for each of the three taxes/subsidies, 0.25 for the correct system.

IV (7.5 points)

a) (1.25 points) Find all the Nash equilibria for the following game, that we will call Game 1:

1 | 2	L	R
U	2,-1	1,1
D	0,0	-1,2

Since the two players have strictly dominant strategies (U for 1 and R for 2), there is a unique Nash equilibrium (U,R).

Grading: 0.75 for the justification, 0.5 for the NE.

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b) (2.25 points) Find all the Nash equilibria for the following game, that we will call Game 2:

1 | 2	L	R
U	0,1	1,0
D	2,0	0,2

Allowing for mixed strategies, we have

		q	1-q
	1 | 2	L	R
p	U	0,1	1,0
1-p	D	2,0	0,2

The payoff of U for 1 is 0.q+1.(1-q)=1-q
The payoff of D for 1 is 2q+0.(1-q)=2q
U is the unique best response if 1-q>2q i.e. if q<1/3
1's BR is therefore to set p=1 if q<1/3, p=0 if q>1/3, and p in [0,1] if q=1/3.

The payoff of L for 2 is p
The payoff of R for 2 is 2(1-p)=2-2p
L is the unique best response if p>2-2p i.e. if p>2/3
2's BR is to set q=1 if p>2/3, q=0 if p<2/3, and q in [0,1] if p=2/3.

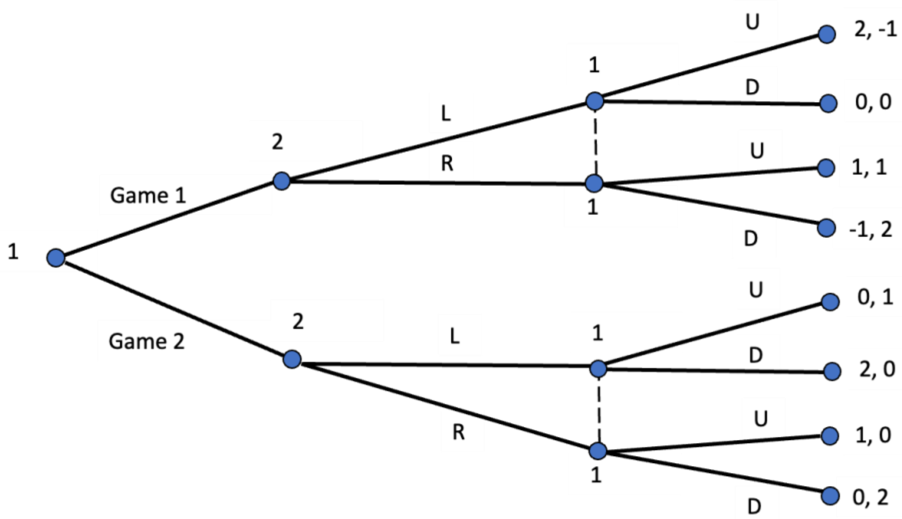
Nash equilibria are characterized by (p=2/3, q=1/3).

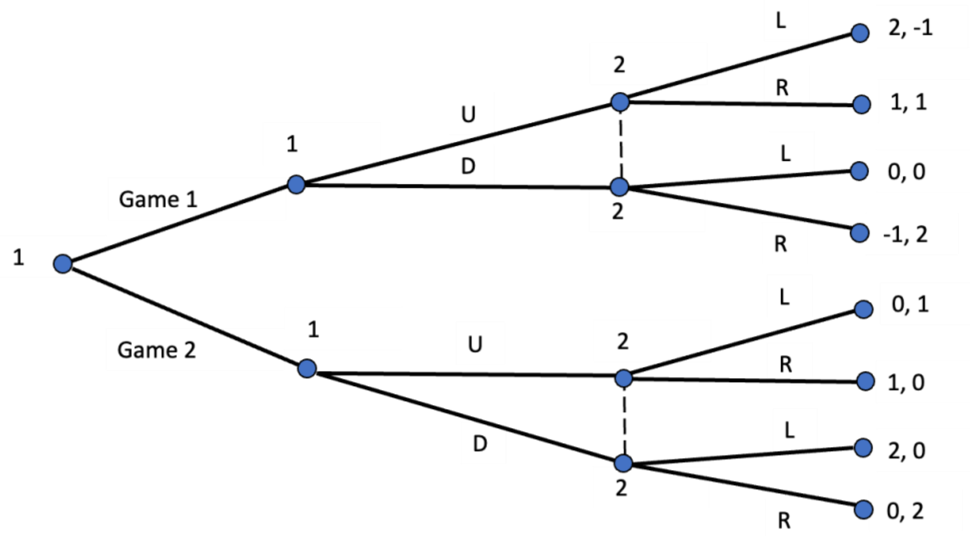
Grading: 0.75 for the best responses of 1, 0.75 for the best responses of 2, 0.75 for the correct conclusion.

c) (4 points) Consider now the "enlarged game" where the first move is Person 1's choice between Game 1 and Game 2.

   i. (1 point) Represent the enlarged game in extensive form.

   There are two possible representations, since Games 1 and 2 are simultaneous:





Note that Game 1 and Game 2 can only be played if player 2 knows which game she is playing; otherwise, there is a different game that player 2 would play, having to set the same L or R for the two matrices (and we would no longer have Game 1 or Game 2).

Grading: 0.5 for understanding the structure of the game (including information), 0.5 for the representation of the two subgames

ii. (0.75 points) How many pure strategies does player 1 have in this game?

Player 1 has 8 pure strategies

Grading: 0.75 for the right conclusion.

iii. (0.75 points) Identify all the subgames in this game.

3 subgames: one after 1 plays 'Game 1', one after 1 plays 'Game 2', and the whole game.

Grading: 0.25 for partial identification of correct subgames; 0.75 for the correct identification of all subgames.

iv. (1.5 points) Find all the SPNE for this game.

We know the NE of Game 1: (U,R); and of Game 2: (p=2/3, q=1/3). At the beginning of the enlarged game, player 1 chooses Game 1 to achieve a payoff of 1 instead of 2/3. The unique SPNE is therefore ((Game 1, U, p=2/3), (R, q=1/3)).

Grading: 0.5 for NE of subgames, 0.5 for the conclusion about 1's move at the beginning of the game, 0.5 for the correct SPNE.