Advanced Microeconomics

Spring 2023
Midterm Exam

Paulo P. Côrte-Real   Nuno Gomes

1. You have a total of 80 minutes (1 hour and 20 minutes) to solve the exam.
2. The use of calculators is not allowed.
3. If you need additional space to answer a question, you can use the back *of the same page*.

Read each question carefully. Good luck!

I (4.5 points)

Consider a pure exchange economy with two agents, A and B, with preferences over two goods given by $u_A(x_A, y_A) = x_A(1 + y_A)$ and $u_B(x_B, y_B) = x_B y_B$.
At the initial endowment $\omega_x^A = 0$, $\omega_y^A = 2$, $\omega_x^B = 2$ and $\omega_y^B = 0$.

a) (2.5 points) Find the Walrasian Equilibrium.

This question could be answered in several ways, among which:

1) Find explicitly the demands of each agent, namely that $(x_A^*, y_A^*) = \left(\frac{3}{2}\frac{p_y}{p_x}, \frac{1}{2}\right)$ and that $(x_B^*, y_B^*) = \left(1, \frac{p_x}{p_y}\right)$ and then conclude that the price ratio for which markets clear is $\frac{p_x}{p_y} = \frac{3}{2}$, with corresponding allocation $(x_A, y_A, x_B, y_B) = \left(1, \frac{1}{2}, 1, \frac{3}{2}\right)$.

2) Find the contract curve given by $y_A = \frac{3}{2}x_A - 1$ for $x_A \geq \frac{2}{3}$ and $y_A = 0$ for $x_A < \frac{2}{3}$ and argue that equilibrium must be on contract curve based on First Welfare Theorem since both agents have weakly monotonic preferences. Conclude that no equilibrium can exist when $x_A < \frac{2}{3}$ because both agents demand strictly positive quantities of y. Use the demand of agent B to conclude that $x_B^* = 1$ and so $x_A^* = 1$ and therefore $y_A^* = \frac{1}{2}$ and so $y_B^* = \frac{3}{2}$. Given that at equilibrium the price ratio equals the MRS of the two agents, and $MRS_{x,y}^A = \frac{1+y_A}{x_A}$, conclude $\frac{p_x}{p_y} = \frac{3}{2}$

The Walrasian Equilibrium is $\left(\frac{p_x^*}{p_y^*}, x_A^*, y_A^*, x_B^*, y_B^*\right) = \left(\frac{3}{2}, 1, \frac{1}{2}, 1, \frac{3}{2}\right)$

Grading: 1 point for justification; 0.75 points for market clearing; 0.375 points for equilibrium price ratio; 0.375 points for equilibrium allocation.

NUMBER: NAME:

b) (1 point) Is the Walrasian Equilibrium Pareto Efficient? Justify.

Because both agents have weakly monotonic preferences, we can apply the First Welfare Theorem and conclude that the Walrasian Equilibrium must be Pareto Efficient.

Grading: 0.5 points to mention agents have weakly monotonic preferences; 0.5 to mention First Welfare Theorem and apply it.

c) (1 point) Consider that agent B instead has preferences represented by $v_B = 4x_By_B + 2$. Without any additional calculations, would this change the Walrasian Equilibrium? Justify.

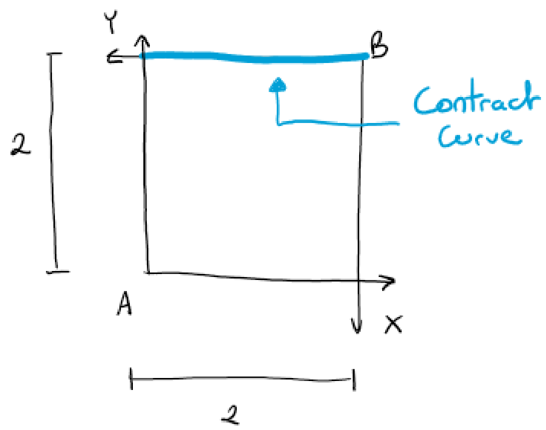
Since v_B is just a positive linear transformation of u_B it represents the same preferences as those under u_B, therefore the solutions for agent B to the utility maximization problem will be exactly the same and so the demand functions will be equal for any prices. Therefore, the Walrasian Equilibrium will be unchanged.

Grading: 0.5 points for mentioning that v_B represents the same preferences as u_B (with justification); 0.25 points for justifying that demands/excess demands will stay the same for all price ratios; 0.25 for concluding that the Walrasian Equilibria will not be changed.

II (5.25 points)

Consider a pure exchange economy with two agents, A and B, with preferences over two goods given by $u_A(x_A, y_A) = x_A + y_A$ and $u_B(x_B, y_B) = 2x_B - y_B$. Consider that at the initial endowment $\omega_x^A = 1$, $\omega_y^A = 1$, $\omega_x^B = 1$ and $\omega_y^B = 1$.

a) (1.25 points) Find the analytical expression of the contract curve and represent it graphically in the Edgeworth Box.



The contract curve is given by $y_A = 2$ (or $y_B = 0$).

Grading: 0.625 points for graphical representation of the contract curve; 0.625 points for analytical expression.

NUMBER: NAME:

b) (2 points) Show that all Pareto Efficient combinations of utility can be given by $u_B = 8 - 2u_A$. For which values of u_A is this expression valid?

Because on the contract curve we have that $y_A = 2$ (or $y_B = 0$), we can rewrite the utilities as $u_A = x_A + 2$ and $u_B = 2x_B$. Moreover, given the feasibility constraints it follows $u_B = 2(2 - x_A)$, we can then relate u_A and u_B by noting that $x_A = u_A - 2$ so that $u_B = 2(2 + 2 - u_A)$ and finally reach $u_B = 8 - 2u_A$. Note that the minimum value of u_A occurs when $x_A = 0$ so that $u_A = 2$ and the maximum value of u_A occurs when $x_A = 2$ so $u_A = 4$, therefore the expression is valid for values $u_A \in [2; 4]$

Grading: 0.5 points for efficiency; 0.5 for feasibility; 0.5 points for relating utilities; 0.5 points for range of u_A

c) (2 points) Find the socially optimal allocation under a utilitarian social welfare function.

We wish to solve the problem:

$$\max_{\{u_A, u_B\}} W = u_A + u_B$$
$$s.t. u_B = 8 - 2u_A$$

And considering that $u_A \in [2; 4]$. We can rewrite W to simplify the problem:

$$\max_{\{u_A\}} W = 8 - u_A$$

And note that $\frac{dW}{du_A} = -1 < 0$ so that W is strictly decreasing on u_A, so at the optimum it must be set as low as possible. Hence $u_A^* = 2$ which implies $u_B^* = 4$. Given $y_A^* = 2$ (because we must have a Pareto Efficient allocation), it follows that $y_B^* = 0$ and moreover we have that $2 = x_A + 2$ because $u_A = x_A + y_A$. From here we have that $x_A^* = 0$ and $x_B^* = 2$ so the socially optimal allocation is:

$$(x_A^*, y_A^*, x_B^*, y_B^*) = (0,2,2,0)$$

Grading: 0.25 points for formalization of the problem; 0.25 points for simplification of the problem; 1 point for reaching optimal utilities; 0.5 points for reaching optimal allocation.

III

(3 points)

Assume there are 150 students in a class and that their preferences regarding air conditioning (G) and money (x) vary.

For group A, composed of 100 students, the individual utility function is $u_i(x_i, G) = x_i + \ln(G)$

For group B, composed of 50 students, the individual utility function is $u_i(x_i, G) = x_i + 2\ln(G)$

Assume there is a large amount of money available and that air conditioning can be obtained from money according to the production function $G = f(x) = \sqrt{x}$.

a) (1.75 points) Find the Pareto Efficient level of the public good.

Samuelson Condition: $\sum_{i=1}^{150} MRS_{G,x} = MC_G$

Group A: $MRS_{G,x} = \frac{\delta U/\delta G}{\delta U/\delta x} = \frac{1}{G}$ | Group B: $MRS_{G,x} = \frac{\delta U/\delta G}{\delta U/\delta x} = \frac{2}{G}$

$MC_G = 2\mathrm{G}\ (\text{since } x = G^2)$

Applying the Samuelson Condition: $100 \times \frac{1}{G} + 50 \times \frac{2}{G} = 2\mathrm{G} \Leftrightarrow G^* = 10$

Grading: 0.5 for Samuelson condition, 0.25 for MRSA, 0.25 for MRSB, 0.25 for MC, 0.5 for conclusion

b) (1.25 points) How would you tax the agents if you wanted to achieve a unanimous choice of the level you found in a.?

Lindahl Taxation: each agent should be taxed according to her MRS at the efficient level of the public good $t_i = MRS_i(G^* = 10)$. Accordingly, the unitary taxes should be:

$t_A = \frac{1}{10}$ $t_B = \frac{2}{10} = \frac{1}{5}$

Grading: 0.25 for Lindahl taxation, 0.5 for tA, 0.5 for tB.

IV (3 points)

Consider the following game, where a>0 and b>0.

1 | 2	L	R
U	0,0	0,1
M	a,b	1,2
D	1,-1	3,0

a) (1 point) What values of a and b would ensure an equilibrium in weakly dominant strategies?

R would need to be weakly dominant for 2 and therefore b must be smaller or equal to 2.
D would need to be weakly dominant for 1 and therefore a must be smaller or equal to 1.

Grading: 0.5 for each

NUMBER: NAME:

b) (2 points) Let a=2 and b=3. Find <u>all</u> the Nash equilibria of the game.

1 | 2	L	R
U	0,0	0,1
M	2,3	1,2
D	1,-1	3,0

U is never a best response for 1 and will therefore never be played with positive probability at a Nash equilibrium. We can focus on the reduced matrix:

		q	1-q
	1 | 2	L	R
p	M	2,3	1,2
1-p	D	1,-1	3,0

The payoff of M for 1 is 2.q+1.(1-q)=1+q
The payoff of D for 1 is 1.q+3.(1-q)=3-2q
If q>2/3, the unique best response is p=1. If q<2/3, the unique best response is p=0. If q=2/3, the best responses are p in [0,1]

The payoff of L for 2 is 3.p-1(1-p)=-1+4p
The payoff of R for 2 is 2p
L is the unique best response if -1+4p>2p i.e. if p>1/2
2's BR is to set q=1 if p>1/2, q=0 if p<1/2 and q in [0,1] if p=1/2.

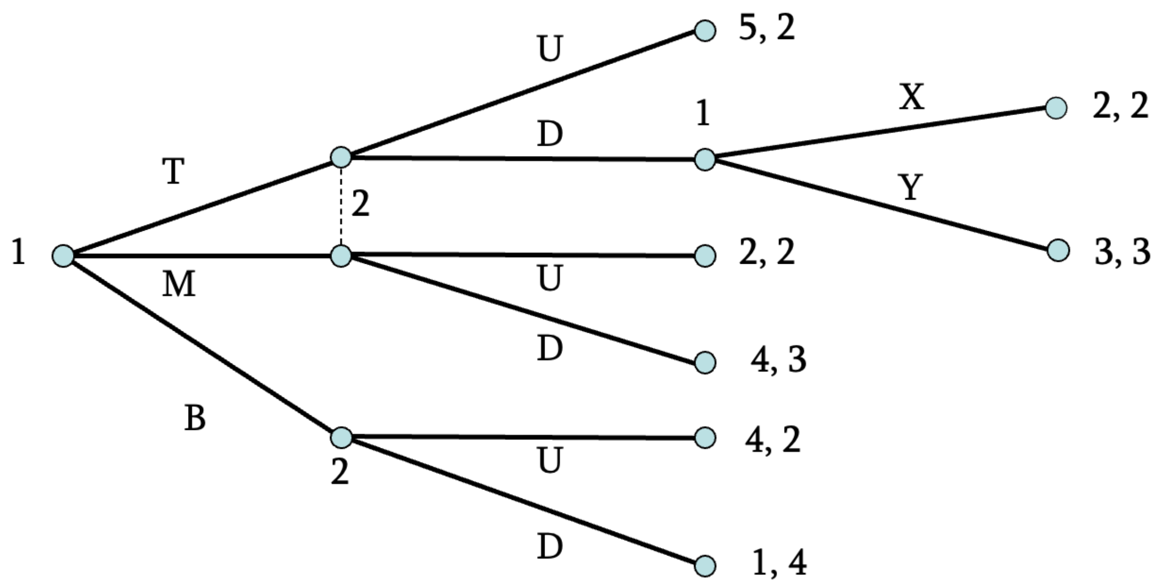
Nash equilibria are characterized by (p=0, q=0), (p=1,q=1) and [p=1/2, q=2/3].

Grading: 0.5 for the elimination of U (with the correct argument), 0.5 for the best responses of 1, 0.5 for the best responses of 2, 0.5 for the correct conclusion.

NUMBER: NAME:

V (4.25 points)

Consider the following dynamic game in extensive form:



a) (0.5 points) Identify the subgames in this game.

3 subgames: one after 1 plays T and 2 plays D; another after 1 plays B; and the whole game.

Grading: 0.25 for partial identification of correct subgames; 0.5 for the correct identification of all subgames.

b) (1 point) How many pure strategies does each player have?

Player 1 has 6 pure strategies, player 2 has 4 pure strategies

Grading: 0.5 for each player.

c) (2.75 points) Find all the SPNE of the game.

Subgame after 1 plays T and 2 plays D: 1 plays Y

Subgame after 1 plays B: 2 plays D

For the whole game, already incorporating these restrictions:

1 | 2	UD	DD
TY	5,2	3,3
MY	2,2	4,3
BY	1,4	1,4

BY is never a best response for 1 and we can eliminate the bottom row. But then UD is never a best response for 2 and therefore the unique NE is (MY, DD).

Grading: 0.5 for the first subgame, 0.5 for the second, 1.25 for the third (including matrix and argument), 0.5 for the correct SPNE.

NUMBER: NAME: