# Advanced Microeconomics

Fall 2022

Midterm Exam

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1. You have a total of 80 minutes (1 hour and 20 minutes) to solve the exam.
2. The use of calculators is not allowed.
3. If you need additional space to answer a question, you can use the back *of the same page*.

Read each question carefully. Good luck!

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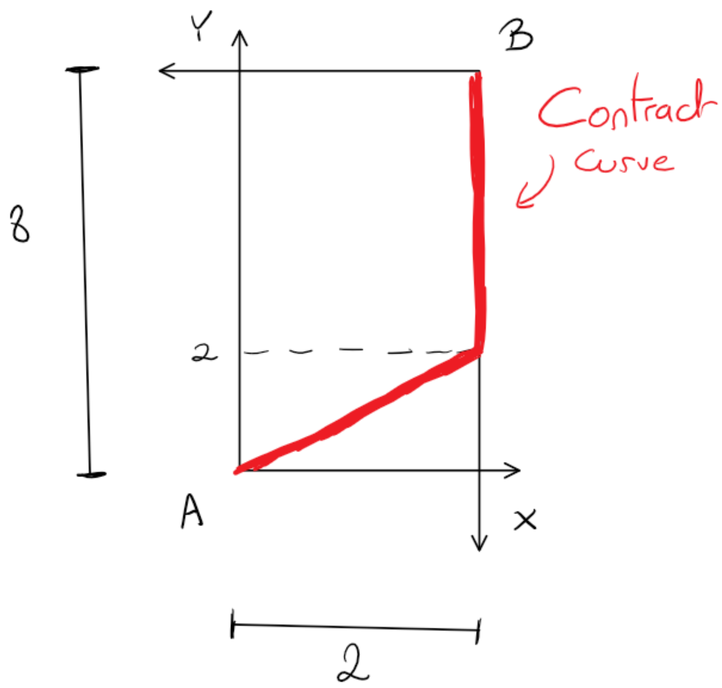
I (4.5 points)

Consider a pure exchange economy with two agents, $A$ and $B$, with preferences over two goods given by $u_A(x_A, y_A) = \min\{x_A, y_A\}$ and $u_B(x_B, y_B) = x_B y_B$. It is known that $\omega_x^A + \omega_x^B = 2$ and that $\omega_y^A + \omega_y^B = 8$.

a) (1.5 points) Find the contract curve and represent it graphically in the Edgeworth Box.

The contract curve is given by: $y_A = x_A$ if $0 \leq y_A < 2$ and $x_A = 2$ if $2 \leq y_A \leq 8$




**Grading:**
**0.45 points for the graphical representation of the segment where $y_A = x_A$**
**0.3 points for the graphical representation of the segment where $x_A = 2$**
**0.45 points for the analytical condition of the segment where $y_A = x_A$**
**0.3 points for the analytical condition of the segment where $x_A = 2$**

NUMBER: NAME:

b) (1.5 points) Show that the UPF is given by $u_B = u_A^2 - 10u_A + 16$.

Allocations must be feasible: $x_A + x_B = 2$ and $y_A + y_B = 8$
Allocations must be on the contract curve: $y_A = x_A$ if $0 \leq y_A < 2$ or $x_A = 2$ if $2 \leq y_A \leq 8$

Case 1: if $y_A = x_A$ (with $0 \leq y_A < 2$)

Then this implies $u_A = \min\{x_A, x_A\} = x_A$
And $u_B = x_B y_B = (2 - x_A)(8 - y_A) = (2 - x_A)(8 - x_A)$

Where we can then replace $x_A$ by $u_A$ to get: $u_B = (2 - u_A)(8 - u_A) = u_A^2 - 10u_A + 16$
(This is valid for $u_A \in [0; 2[$ given that $0 \leq y_A < 2$)

Case 2: if $x_A = 2$ (with $2 \leq y_A \leq 8$)

Then this implies $u_A = \min\{2, y_A\} = 2$ since $y_A \geq 2$
And $x_B = 0$, which means that $u_B = 0$.
So the point $(u_A, u_B) = (2,0)$ is also on the UPF.
This point can also be accommodated by the expression $u_B = u_A^2 - 10u_A + 16$

Therefore, the UPF is given by $u_B = u_A^2 - 10u_A + 16$

**Grading:**
**0.25 points for stating that allocations must lie on contract curve**
**0.25 points for stating feasibility conditions**
**0.75 points for writing an expression relating agents' utilities in case $y_A = x_A$ and $0 \leq y_A < 2$**
**0.25 points for writing an expression relating agents' utilities in case $x_A = 2$ and $2 \leq y_A \leq 8$**

c) (1.5 points) Find the socially optimal allocation under a utilitarian social welfare function.

The utilitarian choice for this economy solves:

$$\max_{\{u_A, u_B\}} W = u_A + u_B$$
$$s.t.\, u_B = u_A^2 - 10u_A + 16$$

Replacing $u_B$ on $W$ we get the simpler problem:

$$\max_{\{u_A, u_B\}} W = u_A^2 - 9u_A + 16$$

Where, given that W is strictly convex, the optimum must be either at the maximum or minimum values of $u_A$, namely: $u_A = 0$ or $u_A = 2$.

If $u_A = 0$, $u_B = 16$ so $W = 16$
If $u_A = 2$, $u_B = 0$ so $W = 2$
Therefore, the optimum must be where $(u_A^*, u_B^*) = (0,16)$
This corresponds to the allocation where $(x_A, y_A, x_B, y_B) = (0,0,2,8)$

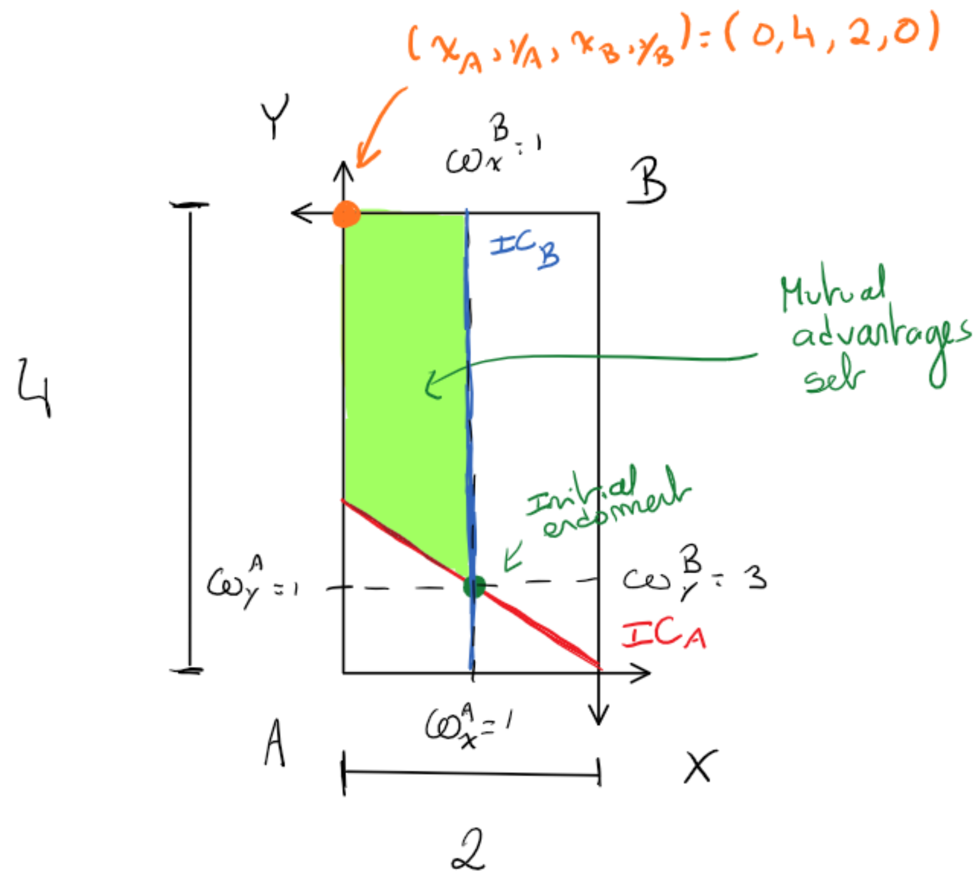
**Grading:**
**0.25 points for formalization of the problem**
**0.25 points for simplification of the problem**
**0.75 points for reaching optimal utilities**
**0.25 points for reaching optimal allocation**

NUMBER: NAME:

II (5.5 points)

Consider a pure exchange economy with two agents, $A$ and $B$, with preferences over two goods given by $u_A(x_A, y_A) = x_A + y_A$ and $u_B(x_B, y_B) = x_B$. Moreover, consider that the initial endowments are given by $\omega_x^A = 1$, $\omega_y^A = 1$, $\omega_x^B = 1$ and $\omega_y^B = 3$.

a) (1 point) Is the allocation $(x_A, y_A, x_B, y_B) = (0,4,2,0)$ in the mutual advantages set? Justify, using an Edgeworth box.



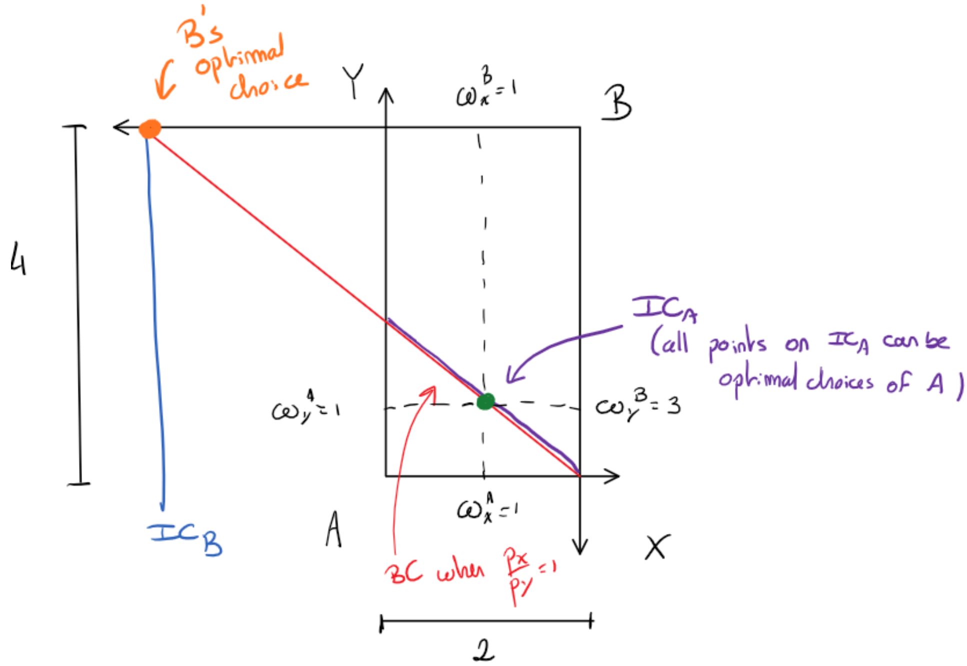

**Grading:**
**0.5 points for the Edgeworth box**
**0.5 points for concluding it is on the mutual advantages set**

NUMBER: NAME:

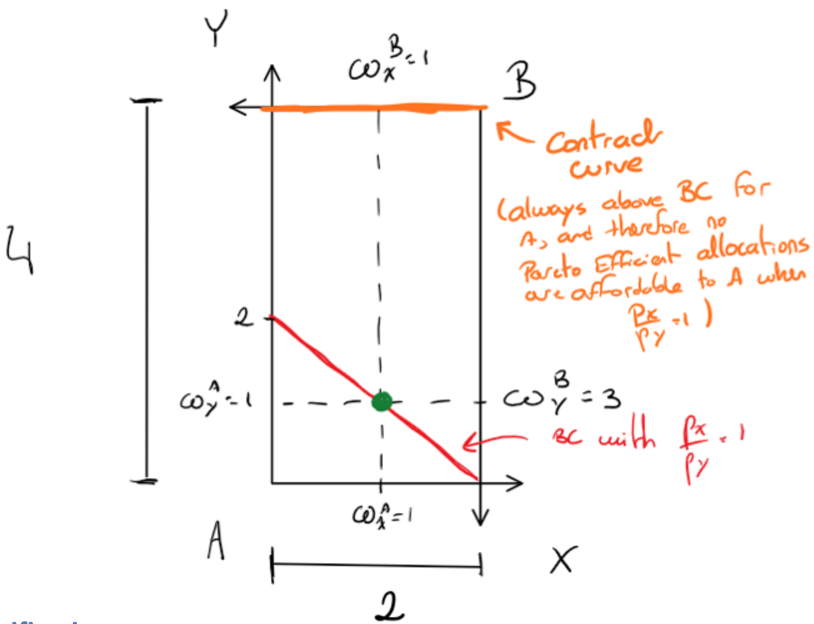
b) (1.5 points) Explain (again using the Edgeworth box) why the equilibrium price ratio $\frac{p_x}{p_y}$ cannot be 1 in this economy.

There are multiple ways to justify why this can't be the equilibrium price ratio. For instance:

1) Use the demands of both agents to conclude that with a price ratio of 1 agent A will consume any bundle such that $x_A + y_A = 2$ but B will wish to consume $x_B^* = 4$ and $y_B^* = 0$, implying agent A would need to consume $x_A^* = -2$ and $y_B = 6$ in equilibrium, which is impossible.




2) Justify through the first welfare theorem that the equilibrium must be on the contract curve, which is given by $y_A = 4$. With a price ratio of 1 agent A's budget constraint will be $x_A + y_A = 2$ which makes any bundle with $y_A = 4$ unaffordable, and hence impossible to reach.




**Grading:**
**1 point for the justification**
**0.5 points for the Edgeworth box**

NUMBER: NAME:

c) (2 points) Find the equilibrium price ratio $\frac{p_x}{p_y}$, concluding that in this case it is greater than 1. Find the associated equilibrium allocation.

We already know no equilibrium exists where $\frac{p_x}{p_y} = 1$ from question b)
If $\frac{p_x}{p_y} < 1$, then $y_A^* = 0$ and $y_B^* = 0$.
This implies $y_A^* + y_B^* = 0 < 4 = \omega_y^A + \omega_y^B$. There is excess supply in this market so there can't be an equilibrium with a price ratio smaller than 1.
In case an equilibrium exists where $\frac{p_x}{p_y} > 1$, we can focus our analysis just on one market since by Walras' Law if one market clears the other must clear as well.

The market for $Y$ clears if:
$$y_A^* + y_B^* = 4 \Leftrightarrow 1 + \frac{p_x}{p_y} = 4 \Leftrightarrow \frac{p_x}{p_y} = 3$$
Moreover, when $\frac{p_x}{p_y} = 3$ we have that $x_A^* = 0$, $y_A^* = 4$, $x_B^* = 2$, $y_B^* = 0$.
The equilibrium price ratio is $\frac{p_x}{p_y} = 3$ and the equilibrium allocation is $(x_A, y_A, x_B, y_B) = (0,4,2,0)$

**Grading:**
**0.3 points for showing no equilibrium exists where $\frac{p_x}{p_y} < 1$**
**0.7 points for justification (0.4 for correct demands + 0.3 points for market clearing condition when $\frac{p_x}{p_y} > 1$)**
**0.5 points for obtaining equilibrium price ratio**
**0.5 points for obtaining equilibrium quantities**

d) (1 point) Without additional calculations, could you state whether the allocation you found in c) is Pareto efficient?

Yes, it is. Both agents have weakly monotonic preferences, so we can apply the first welfare theorem and conclude that since the allocation found in the previous question is part of a Walrasian equilibrium then it must be Pareto efficient.

**Grading:**
**0.5 points for concluding both agents have weakly monotonic preferences**
**0.5 points for mentioning the first welfare theorem and that it implies Walrasian equilibria are Pareto Efficient**

## III
(3 points)

There are two firms Glasco and Dram Ltd who are located within a few miles of each other. Glasco is a local coal-processing plant and Dram is an electronics company producing microchips. There is a problem in that the processing of coal causes air pollution, which is of great detriment to the production of microchips.
The marginal private cost function of processing coal per week to Glasco is 10+0.5Q and the marginal cost imposed on Dram is 5+0.25Q, where Q measures the amount of coal in tonnes. Let Glasco be a price-taker at a price of 30 per ton of coal.

a) (1.75 points) What would be the competitive and socially optimal level of coal processing?

Competitive solution: 30=10+0.5Q and Q=40
Socially optimal: 30=(10+0.5Q)+(5+0.25Q) and Q*=20

*Grading: 0.75 points for the competitive solution, 1 point for the efficient level.*

b) (1.25 points) If the government became alarmed at the problems caused to the microchip company, what level of tax would be needed to reduce the (net) price of coal to the level that is socially optimal?

Pigouvian tax where t=MEC(Q*)=5+0.25Q*=5+0.25*20=10

*Grading: 0.25 for identifying this should be the Pigouvian tax, 1 for the calculation*

## IV (2.5 points)

Find all the Nash equilibria for the following game:

| 1 \| 2 | L | C | R |
|---|---|---|---|
| U | 1,2 | 3,4 | 4,3 |
| M | 7,2 | 3,1 | 7,1 |
| D | 1,1 | 2,5 | 2,3 |

R is never a best response for 2 and will therefore never be played with positive probability at a Nash equilibrium. D is never a best response for 1 and will therefore never be played with positive probability at a Nash equilibrium. We can focus on the reduced matrix:

| | | q | 1-q |
|---|---|---|---|
| | 1 \| 2 | L | C |
| p | U | 1,2 | 3,4 |
| 1-p | M | 7,2 | 3,1 |

The payoff of U for 1 is 1.q+3.(1-q)=3-2q
The payoff of M for 1 is 7q+3.(1-q)=3+4q
It is clear that M will be the unique best response except if q=0.

NUMBER: NAME:

1's BR is therefore to set p=0 if q>0 and p in [0,1] if q=0.

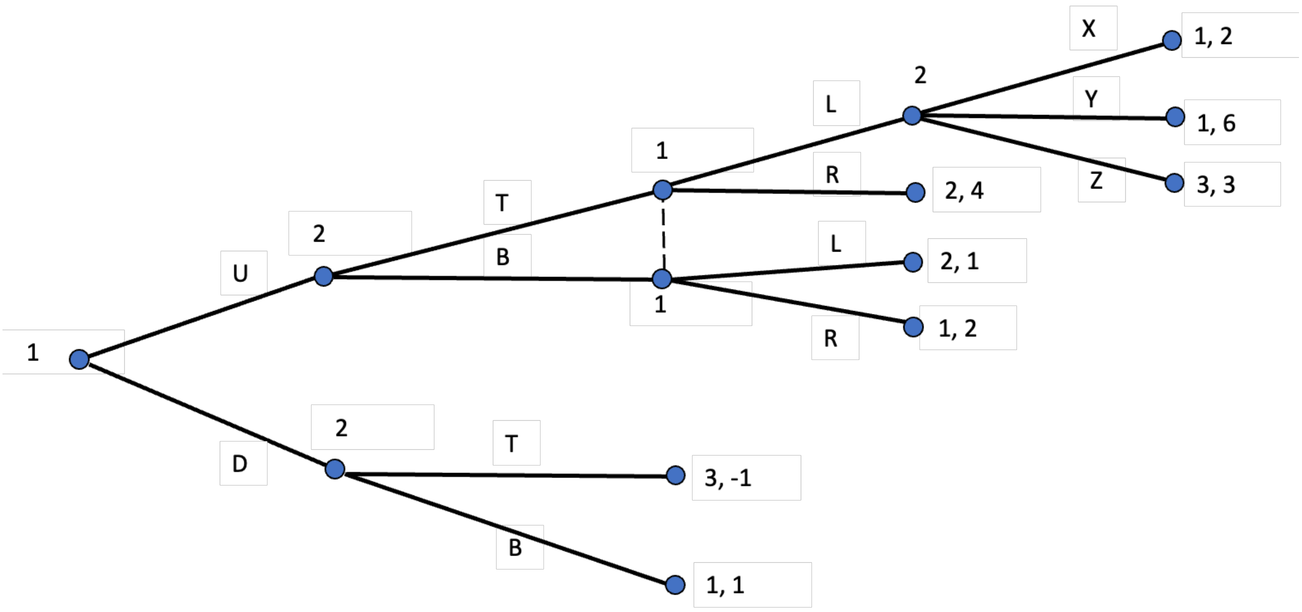
The payoff of L for 2 is 2
The payoff of C for 2 is 4p+1-p=3p+1
L is the unique best response if 2>3p+1 i.e. if p<1/3
2's BR is to set q=1 if p<1/3, q=0 if p>1/3 and q in [0,1] if p=1/3.

Nash equilibria are characterized by (p=0, q=1) and [p>=1/3, q=0].

*Grading: 0.5 for the elimination of D and R, 0.5 for the correct argument, 0.5 for the best responses of 1, 0.5 for the best responses of 2, 0.5 for the correct conclusion.*

V (4.5 points)

Consider the following dynamic game in extensive form:




a) (0.5 points) Identify the subgames in this game.

4 subgames: one after 1 plays U, 2 plays T and 1 plays L; another after 1 plays U; another after 1 plays D; and the whole game.

*Grading: 0.25 for partial identification of correct subgames; 0.5 for the correct identification of all subgames.*

b) (1 point) How many pure strategies does each player have?

Player 1 has 4 pure strategies, player 2 has 12 pure strategies

*Grading: 0.5 for each player.*

c) (3 points) Find <u>all</u> the SPNE of the game.

- Subgame after 1 plays U, 2 plays T and 1 plays L: 2 plays Y

- Subgame after 1 plays D: 2 plays B

- Subgame after 1 plays U:

| | | q | 1-q |
|---|---|---|---|
| | 1 \| 2 | TY | BY |
| p | L | 1,6 | 2,1 |
| 1-p | R | 2,4 | 1,2 |

TY is a strictly dominant strategy for 2 and therefore the unique NE is (R, TY).

- At the beginning of the game, 1 will therefore play U.

The unique SPNE is therefore (UR, TYB).

*Grading: 0.5 for the first subgame, 0.5 for the second, 1 for the third (including matrix and argument), 0.5 for the conclusion about 1's move at the beginning of the game, 0.5 for the correct SPNE.*