PS 5 Extra Exercises

1. Consider the following game in strategic form:

1 \ 2	A	B	C
X	12,7	6,3	6,6
Y	5,2	8,0	3,5
Z	11,6	1,7	8,10

Find all the Nash equilibria of the game.

B is never a best response for agent 2. Therefore, we can eliminate that column from the matrix, since it will never be played in any Nash equilibrium. But then Y is never a best response by player 1 to any of the remaining strategies for 2 – and Y will therefore never be played in any Nash equilibrium. We can therefore also eliminate the Y row from the matrix.

The reduced game is

1 \ 2	A	C
X	12,7	6,6
Z	11,6	8,10

Letting p denote the probability of playing X for player 1 and q denote the probability of playing A for player 2, there are three Nash equilibria in this game: (X,A), (Z,C) and (p=4/5, q=2/3).

2. Consider two firms that produce a homogeneous good whose demand is given by $Q = 500 - 50P$. Firm 1 has a marginal cost equal to 8, whereas firm 2's marginal cost is equal to 6. For the following two cases, calculate the Nash equilibria (price, quantities, and profits):

a) Firms compete in quantities.

The Cournot-Nash equilibrium is q1=0 and q2=100.

b) Firms compete in prices.

In this case, there is no Nash equilibrium in pure strategies.

If the marginal costs were the same, the Nash equilibrium would be for both to sell at a price equal to the marginal cost. If one firm were to set a higher price, the firm would sell nothing. If one firm were to set a lower price, the firm would make a loss.
In this case, however, there is no equilibrium in pure strategies. The market price will always be the lowest of the prices announced by the two firms. At any market price greater or equal to 8, firm 2 would always want to set a slightly lower price to capture the whole market. At any price below 8, firm 1 would make a loss and firm 2 would therefore have the whole market. But then firm 2 would have an incentive to keep raising the price and there is therefore no equilibrium in pure strategies.

3. Consider the following model of price competition. Two firms set prices in a market whose demand curve is given by the equation: $Q = 6 - p$, where p is the lowest of the two prices. If firm 1 sets a lower price, then it supplies all the demand; conversely, the same applies to firm 2. For example, if firms 1 and 2 set prices equal to €2 and €4, respectively, then firm 1 sells 4 units, whereas firm 2 sells 0 units. If the two firms set the same price p, then they each get half the market, that is, they each get $\frac{6-p}{2}$. Suppose that prices can only be quoted in euro-units, such as 0,1,2,3,4,5 or 6 euros. Suppose further that costs of production are zero for both firms.

   (a) Write down the strategic form of this game assuming that each firm cares only about its own profits.

   Answer

1—2	0	1	2	3	4	5	6
0	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
1	(0,0)	(2.5,2.5)	(5,0)	(5,0)	(5,0)	(5,0)	(5,0)
2	(0,0)	(0,5)	(4,4)	(8,0)	(8,0)	(8,0)	(8,0)
3	(0,0)	(0,5)	(0,8)	(4.5,4.5)	(9,0)	(9,0)	(9,0)
4	(0,0)	(0,5)	(0,8)	(0,9)	(4,4)	(8,0)	(8,0)
5	(0,0)	(0,5)	(0,8)	(0,9)	(0,8)	(2.5,2.5)	(5,0)
6	(0,0)	(0,5)	(0,8)	(0,9)	(0,8)	(0,5)	(0,0)

   (b) Show that the strategy of setting a price of €5 (weakly) dominates the strategy of posting a price of €6. Does it strongly dominate as well?

   Answer

   It is enough to say that at least for one alternative, the firm is better off picking 5 compared to 6. That is the case, if the other firms chooses from 0 to 4, the payoff is equal, but if the other firm chooses 5 or 6 the payoff are 2.5 and 5 respectively, both greater than 0, which would be the payoff if 6 was chosen.

   (c) Are there any other (weakly) dominated strategies for firm 1? Explain.

   Answer

   3 weakly dominates 4,5 and 6. Also 4 w.d. 5 and 6. The reasoning is the same that was applied in (b). Also 2 w.d. 4 and 5 and 6, but not 3. 1,2,3,4,5 w.d. 0.

   (d) Is there a dominant strategy for firm 1? Explain.

   Answer

   No, because the candidate, 1, has no strictly greater payoff if the other firm chooses a price of zero. Recall that for strict dominance, the alternative should deliver a payoff strictly greater than every other strategy, no matter what the other firm does.

   (e) Say that if both firms share the price, they do not share the market, but instead firm 1 keeps the whole market. What changes? Discuss.

Answer

1—2	0	1	2	3	4	5	6
0	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
1	(0,0)	(5,0)	(5,0)	(5,0)	(5,0)	(5,0)	(5,0)
2	(0,0)	(0,5)	(8,0)	(8,0)	(8,0)	(8,0)	(8,0)
3	(0,0)	(0,5)	(0,8)	(9,0)	(9,0)	(9,0)	(9,0)
4	(0,0)	(0,5)	(0,8)	(0,9)	(8,0)	(8,0)	(8,0)
5	(0,0)	(0,5)	(0,8)	(0,9)	(0,8)	(5,0)	(5,0)
6	(0,0)	(0,5)	(0,8)	(0,9)	(0,8)	(0,5)	(0,0)

Apply the same reasoning. Note that the game is not symmetric anymore. Best Reply ($br_1()$) now is $br_1(6) = br_1(5) = br_1(4) = br_1(3) = 3$, $br_1(2) = 2$, $br_1(1) = 1$ and $br_1(0) = s$. For 2 $br_2(6) = br_2(5) = br_2(4) = 3$, $br_2(3) = 2$, $br_2(2) = 1$, $br_2(1) = br_2(0) = s$. Note that s represents any possible price, so the agent is indifferent between which price to choose. We have the same solutions than above. For both situations do the graph of best reply functions and find that (0,0) and (1,1) are equilibrium candidates.

4. Two swimmers - Evans and Smith - are to participate in a runoff. Each athlete has the option of using a performance-enhancing steroid (s) or not using it (n) before they meet. The two swimmers are equally good, and each has a 50% chance of winning, everything else being equal, that is, if neither uses steroids or they both do. If only one swimmer uses steroids, then he will win.

   (a) Without any IOC[1] intervention, write down the payoff matrix, assuming that the payoff of winning is 1 and the payoff to losing is -1. Also compute the expected payoff for each swimmer.

   Answer

   The expected payoff are the easiest to compute at this stage:

$$\frac{1}{2}1 + \frac{1}{2}(-1) = 0$$

   and the matrix is:

Evans/Smith	s	n
s	0,0	1,-1
n	-1,1	0,0

   (b) Now, assume the IOC decides to intervene. Suppose that the IOC can test only one swimmer. Its choices being either to (a) test Evans, (b) test Smith or (c) use a mixed strategy and test Evans with probability p (and Smith with probability $(1-p)$). If the IOC catches a cheating swimmer, it improves its reputation, getting a payoff of 1. If it does not, then gets a payoff of 0. If the swimmer tests positive, has a penalty of $-(1+b)$, where $b > 0$, and the other swimmer wins the race automatically. Derive the payoff matrices for the three players.

   Answer

Evans/Smith	s	n	Evans/Smith	s	n
s	-1-b,1,1	-1-b,1,1	s	1,-1-b,1	1,-1,0
n	-1,1,0	0,0,0	n	1,-1-b,1	0,0,0

   Where the first table is the IOC testing Evans, and the second one is the IOC tests Smith.

   (c) Find the pure strategy Nash equilibria, if they exist.

   Answer

   If they test Evans for sure, then we are in the first matrix. It is easy to see that Evans has a dominant strategy -n- and so does Smith -s. Exactly the opposite is true if Smith is tested for sure. So the outcome is that the swimmer who knows she will be tested stays away from drugs, but the other swimmer uses steroids.

   (d) Look for mixed strategies, assuming that $p = 0.5$.

   Answer

   Ok, so if $p = 0.5$, we call γ the probability that Evans takes the drug, and δ the probability of Smith taking the drug. So Evans is going to set the probability of taking the drug such that Smith is indifferent between taking the drug or not.

If Evans plays n, and Smith plays s, then Evans gets $0.5(-1) + 0.5(1) = 0$. If Smith plays n, then Evans gets $0.5(0) + 0.5(0) = 0$. So Evans gets always a payoff of zero no matter what Smith can do. If Smith plays s, given that Evans plays s, are $0.5(1)+0.5(-1-b) = \frac{-1}{2}b$, and if Evans plays n, then he gets $0.5(1)+0.5(-1-b) = \frac{-1}{2}b$. So we have, that no matter what Evans do, Smith playing s is always worse. Note that this is symmetric, so we have that playing n for both is dominant for both when the IOC sets a random testing for both swimmers.

PS 5 Extra Exercises

1.

b) 4 subgames.
c) SPNE found through backward induction (ooa, t);
d) NE in pure strategies: (eta, a), (eaa, a), (ott, t), (ota, t), (oto, t), (oat, t), (oaa, t), (oao, t), (oot, t), (ooa, t), (ooo, t).

2. In 1962, the Soviet Union installed nuclear missiles in Cuba. When the US found out, President Kennedy discussed the following options:

   - do nothing,
   - air strike on the missiles,
   - a naval blockade on Cuba.

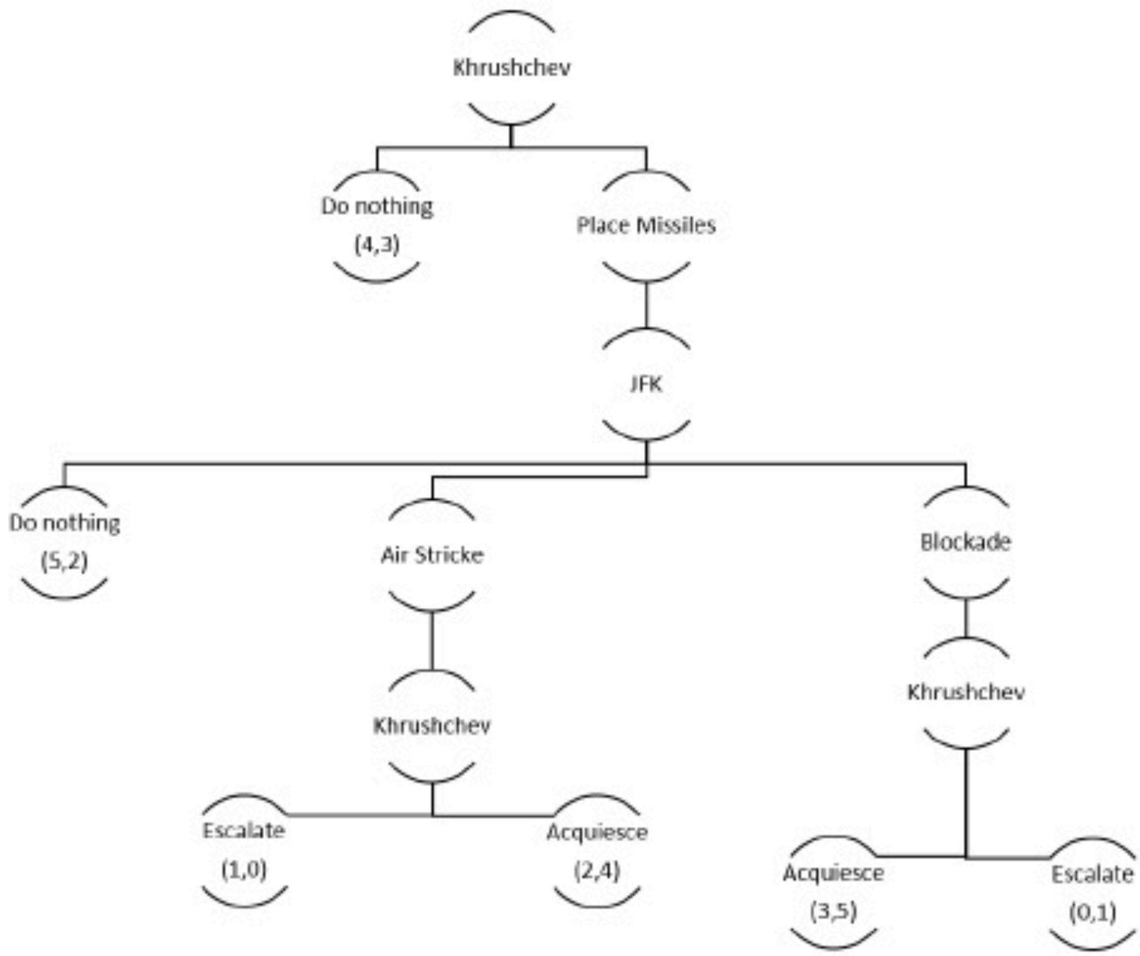
   JFK decided on the naval blockade. Negotiations ensued, and Khrushchev threatened to escalate the situation; both sides believed that nuclear war was a possibility. Finally, the Soviet Union agreed to remove the missiles if the United States agreed not to invade Cuba.

   First, Khrushchev must decide whether to place the missiles in Cuba or not. If the missiles are in place, JFK must decide on his 3 options. For the last two, Khrushchev must decide if acquiesce or escalate.

   If Khrushchev does nothing, the payments are (4,3). If he places the missiles, and JFK responds with airstrike, then K can acquiesce and the payoffs are (2,4), or escalate with resulting payoffs (1,0). If JFK responds with the Blockade, K can acquiesce (3,5) or escalate (0,1). If JFK does nothing, then the payoffs are (5,2).

   (a) Draw the Tree of this game.

   Answer



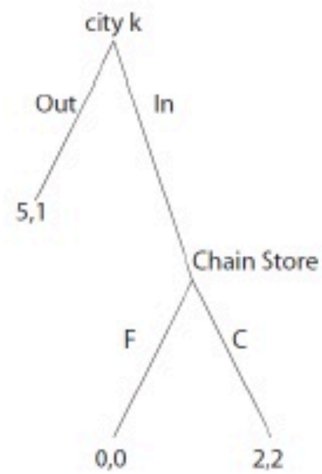
(b) Solve this game (Use BI).

Answer

Using backward induction, we can have They payoffs fro JFK are (5,2) for do nothing, (2,4) for air strike and (3,5) for blockade. From there, we have that hi is going to choose blockade, and then for Khrushchev his payoffs are (4,3) for doing nothing and (3,5) for putting the missiles. Then the solution is for Khrushchev to do nothing.

3. A chain store has branches in K cities, and in each city, $k = 1, ..., K$ there is a competitor. In period k, the competitor in city k enters, the chain store must decide whether to fight or cooperate. This is a game of perfect information, with the payoffs in each city given in the figure. Firm k cares only about the actions taken in its city, but the chain store's payoff is the sum of the payoffs it generates in each city.

The following is the extensive form for the city k:



(a) In which paths of the game do you find Nash Equilibria?

Answer

In every path of the game in which the outcome in any period is either *out* or *(in, C)*.

(b) What is the unique subgame perfect equilibrium?

Answer

The SPNE is *(in, C)*. This is because the firm will enter, because if it enter, knows that the incumbent will cooperate. If it stays out, it could do better entering passing from a payoff of 1 to 2.

(c) For many cities, would it not be better for the incumbent to signal to be tough, and play F?

Answer

This has the logic that if the incumbent makes the entrant to strongly believe that he is going to fight, promising zero payoff in case of entrance and making him to compare 1 versus 0, and then staying out. This would be true, and the idea behind is that for any backward induction to be a SPNE solution, is that the game is finite. If $k \to \infty$, then clearly we are not satisfying this condition, and we would need some temporal dimension and discount rate to compute which would be the optimal strategy.