1. Consider the following game in strategic form:

1\2	D E		F
А	1,0	3.1	1,1
В	1,1	3,2	0,1
С	2,2	3,3	0,2

- a) Identify all pairs of strategies where one weakly dominates the other.
- b) Do the same for strategies where one strongly dominates the other.
- c) Does the game have an equilibrium in dominant strategies?
- 2. Consider the following game in strategic form:

1\2	А	В	С	D
а	1,0	3,1	2,2	8,-1
b	2,4	7,1	-1,3	6,0
с	3,6	8,0	0,5	4,1
d	0,0	6,1	1,6	3,2

- a) Are there strategies that cannot be part of a Nash equilibrium of this game? If so, which and why? Represent the reduced game obtained by eliminating these strategies.
- b) Find all pure and mixed strategy Nash equilibria of the reduced game.

3. Cournot model: Two firms compete in quantities  $q_i$  of a homogeneous good, setting quantities at the same time. The inverse demand is P = a - bQ, where  $Q = q_1 + q_2$ . Both firms have a constant marginal cost equal to c.

- a) What kind of game is this?
- b) Can you represent it in a strategic form?
- c) What are the firms best responses?
- d) What is the Nash equilibrium?
- e) Consider a market with inverse demand given by P = 120 4Q and a constant marginal cost equal to 20.
  Determine the Nash equilibrium.

## **Extra Exercises**

1. Consider the following game in strategic form:

1\2	Α	В	С
Х	12,7	6,3	6,6
Y	5,2	8,0	3 <i>,</i> 5
Z	11,6	1,7	8,10

Find <u>all</u> the Nash equilibria of the game.

Consider two firms that produce a homogeneous good whose demand is given by Q = 500 – 50P. Firm 1 has a marginal cost equal to 8, whereas firm 2's marginal cost is equal to 6. For the following two cases, calculate the Nash equilibria (price, quantities, and profits):

- a) Firms compete in quantities.
- b) Firms compete in prices.
- 3. Consider the following model of price competition. Two firms set prices in a market whose demand curve is given by the equation: Q = 6 p, where p is the lowest of the two prices. If firm 1 sets a lower price, then it supplies all the demand; conversely, the same applies to firm 2. For example, if firms 1 and 2 set prices equal to 2 and 4, respectively, then firm 1 sells 4 units, whereas firm 2 sells 0 units. If the two firms set the same price p, then they each get half the market, that is, they each get (6-p)/2. Suppose that prices can only be quoted in euro-units, such as 0,1,2,3,4,5 or 6 euros. Suppose further that costs of production are zero for both firms.
- a) Write down the strategic form of this game assuming that each firm cares only about its own profits.
- b) Show that the strategy of setting a price of 5 (weakly) dominates the strategy of posting a price of 6. Does it strongly dominate it as well?.
- c) Are there any other (weakly) dominated strategies for firm 1? Explain.
- d) Is there a dominant strategy for firm 1? Explain.
- e) Say that if both firms share the price, they do not share the market, but instead firm 1 keeps the whole market. What changes? Discuss.
- 4. Two swimmers Evans and Smith are to participate in a runoff. Each athlete has the option of using a performance-enhancing steroid (s) or not using it (n) before they meet. The two swimmers are equally good, and each has a 50% chance of winning, everything else being equal, that is, if neither uses steroids or they both do. If only one swimmer uses steroids, then he will win.
- a) Without any International Olympic Committee intervention, write down the payoff matrix, assuming that the payoff of winning is 1 and the payoff to losing is -1. Also compute the expected payoff for each swimmer.

Now, assume the IOC decides to intervene. Suppose that the IOC can test only one swimmer. Its choices being either to (a) test Evans, (b) test Smith or (c) use a mixed strategy and test Evans with probability p (and Smith with probability (1 - p)). If the IOC catches a cheating swimmer, it improves its reputation, getting a payoff of 1. If it does not, then gets a payoff of 0. If the swimmer tests positive, has a penalty of -(1 + b), where b > 0, and the other swimmer wins the race automatically. Derive the payoff matrices for the three players.

- b) Find the pure strategy Nash equilibria, if they exist.
- c) Look for mixed strategy NE, assuming that p = 0.5.