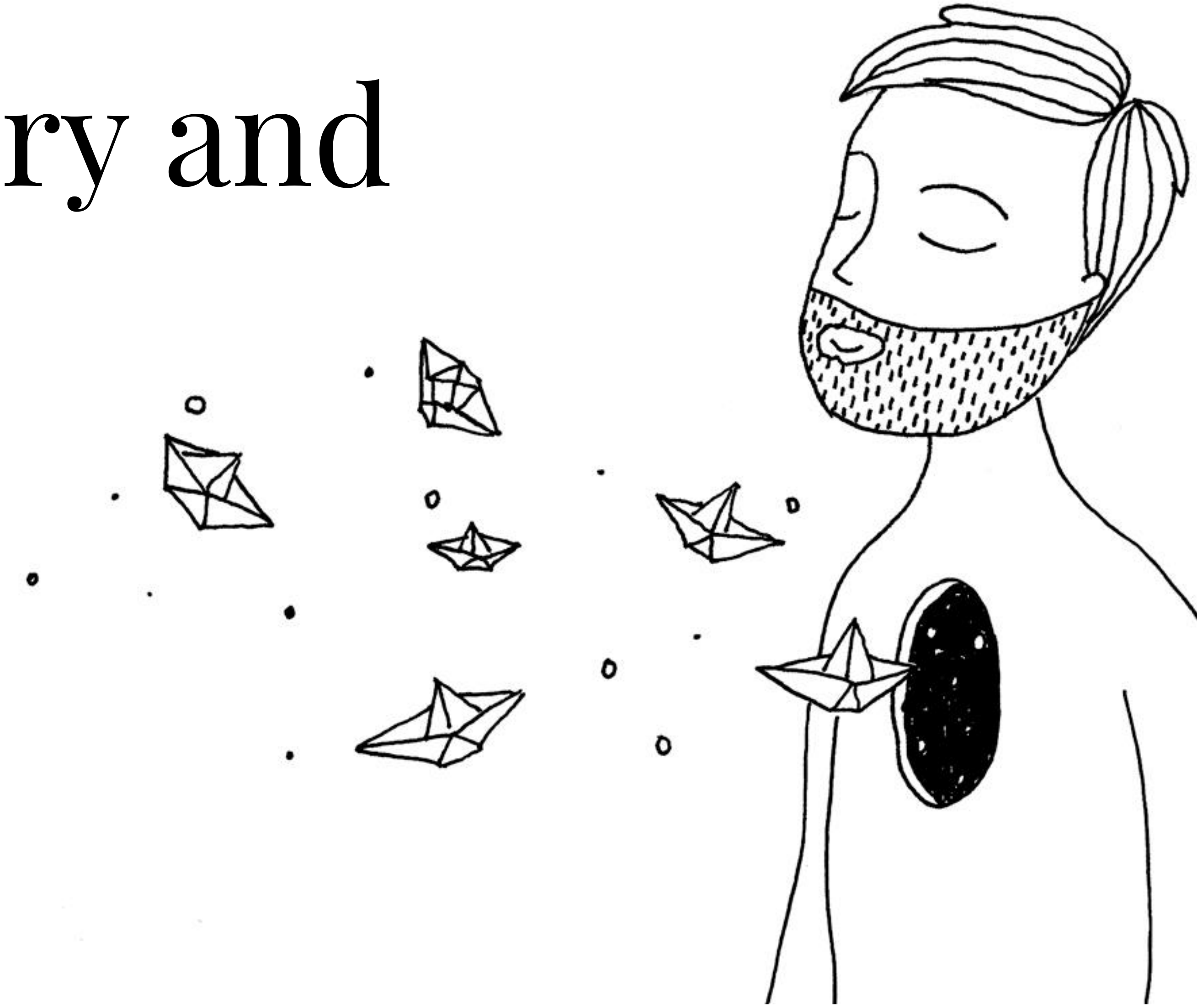
Video Lecture

Portfolio Theory and the CAPM



Advanced Financial Management

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Overview

- Efficient portfolio of risky stocks
- Efficient portfolio of risky stocks and one risk-free asset
- CAPM
- Applications of the CAPM

Efficient portfolio of risky stocks

Modern Portfolio Theory (MPT)

- The founding father of MPT is 1990 Nobel Prize winner Harry Markowitz

- Main question: How much of your wealth to invest in each available asset?
  - Or: How to optimally combine (a large number of) assets in a portfolio?

- Main result:

  Diversification is a "free lunch": the opportunity to reduce risk without sacrificing expected return!

Choosing an efficient portfolio

How can an investor choose an efficient portfolio?

Assumptions

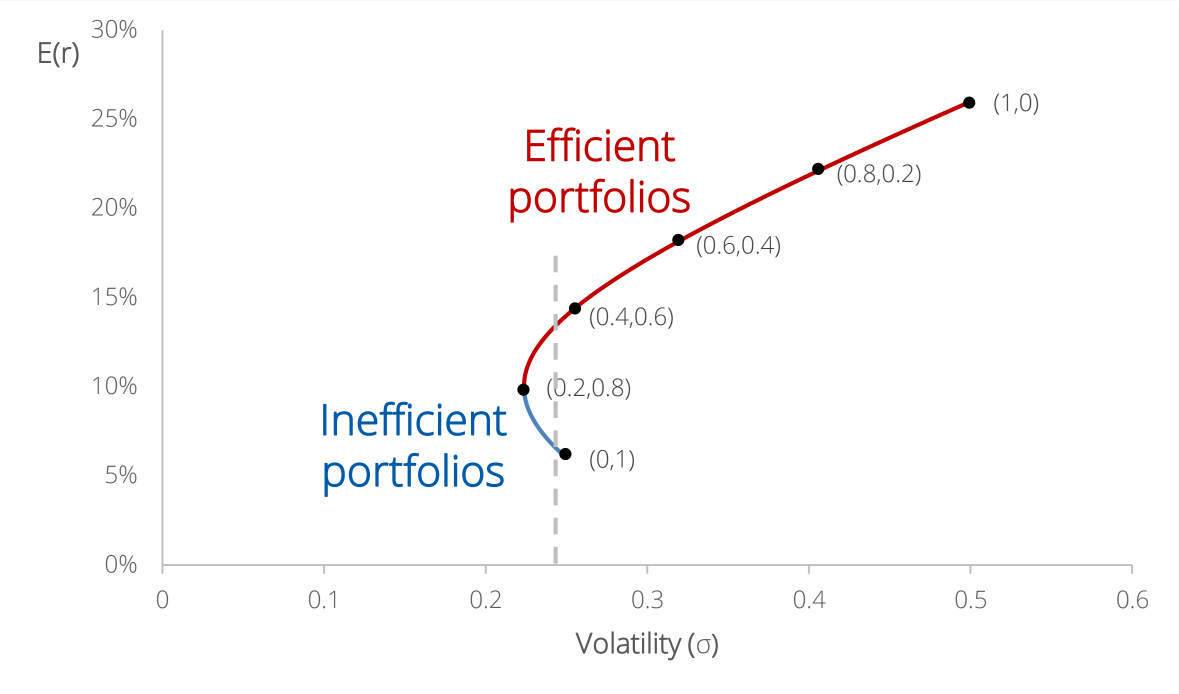
- When comparing 2 portfolios of equal expected return, investors prefer the portfolio with the smallest return standard deviation (or variance).
- When comparing 2 portfolios of equal return standard deviation, investors prefer the one with the higher expected return.
- Investors like reward (mean return) and dislike risk (standard deviation or variance).

A smart investor might pick the level of expected return (s)he wants, and then build the diversified portfolio that has the target expected return but the smallest level of risk (i.e. variance).

Mean-variance efficient portfolio

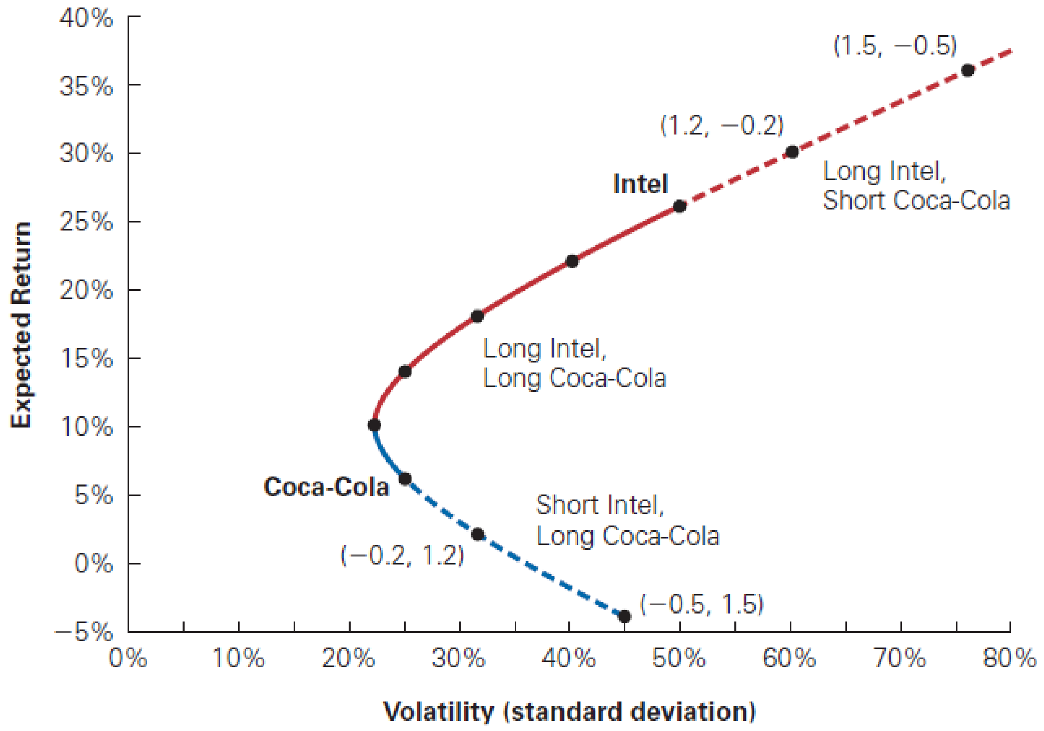
Efficient frontier with 2 securities

- Investor believes the stocks are uncorrelated
- How should the investor choose how much to invest in each stock?*
  - **Inefficient portfolio:** a portfolio which is worse than another portfolio in terms of both E(r) and volatility.
  - **Efficient portfolio:** it has a higher expected return for a given level of volatility than all other portfolios.



* In parentheses are the weights for Intel and Coca-Cola, respectively.

Efficient frontier with short sales



- If we allow short sales, you can obtain an even wider set of risk-return opportunities.
- Aggressive investors may want to be in the efficient frontier taking a long position in Intel and a short position in Coca-Cola

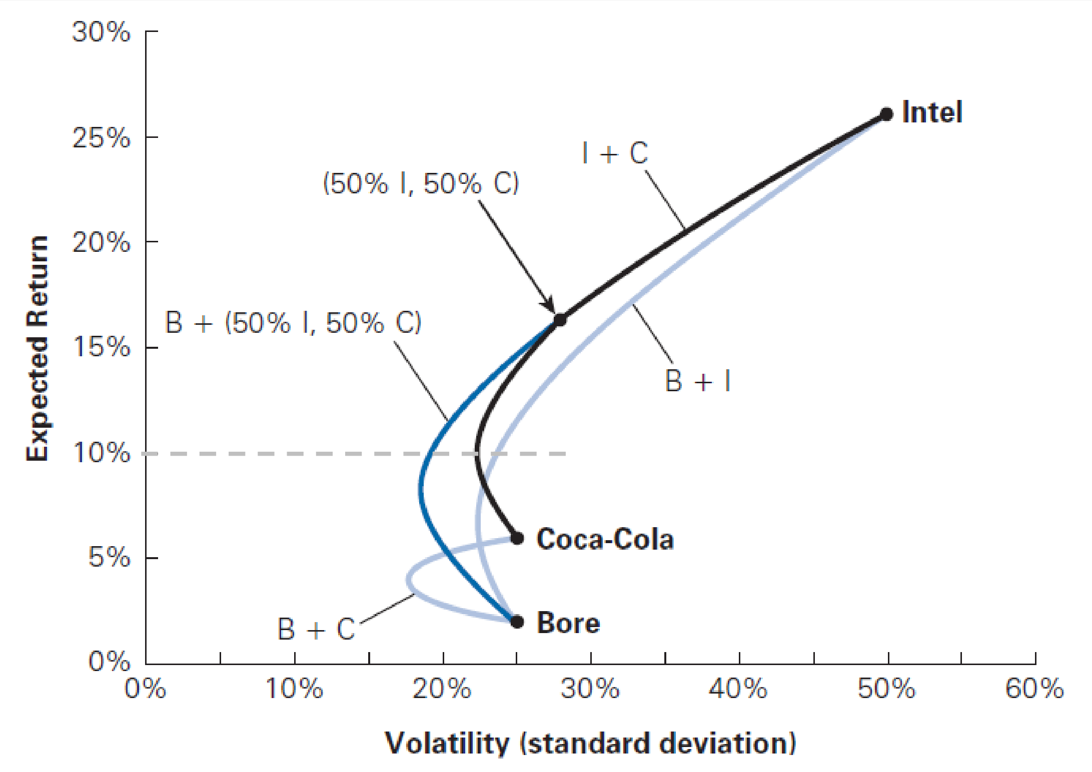
Mean-variance optimization

We will take the following steps:

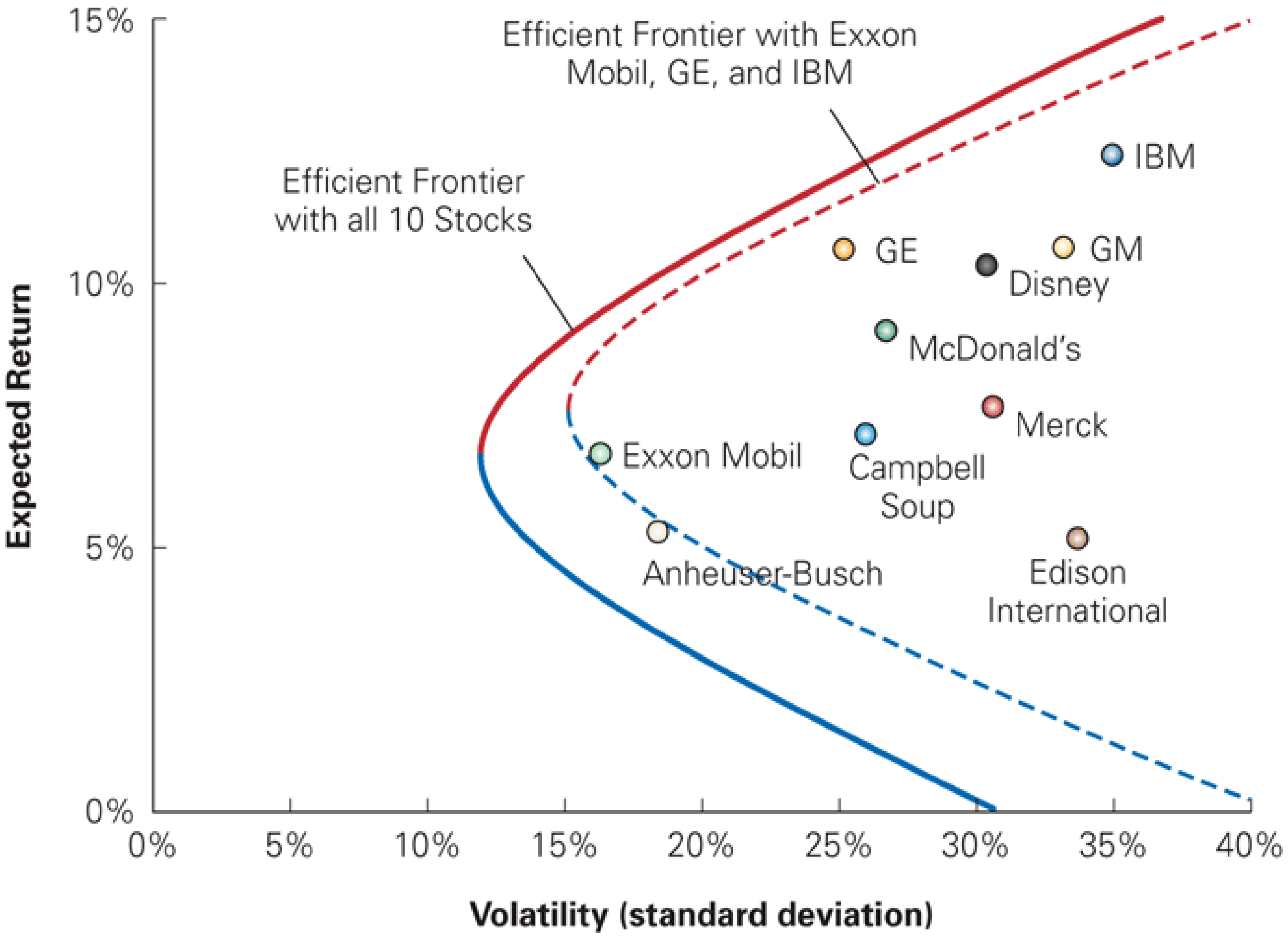
1. Derive the feasible set: the set of portfolio risk and return pairs that it is possible to generate from a given set of securities.
2. Identify the portfolio frontier: this is the set of portfolios with smallest risk for each level of expected return.
3. Identify the efficient frontier: this is the set of portfolios with the highest expected return for each level of risk.
4. Identify the optimal portfolio: it must lie on the portfolio frontier and we use the investor’s indifference curves to identify it.

Efficient portfolios with many stocks

- Lets see what happens when we introduce a third stock, Bore Industries (E(r)=2% and σ=25%), also uncorrelated with Intel and Coca-Cola
- Even though Bore is inferior to Coca-Cola, it adds diversification benefits



Efficient portfolios with many stocks



Steps:

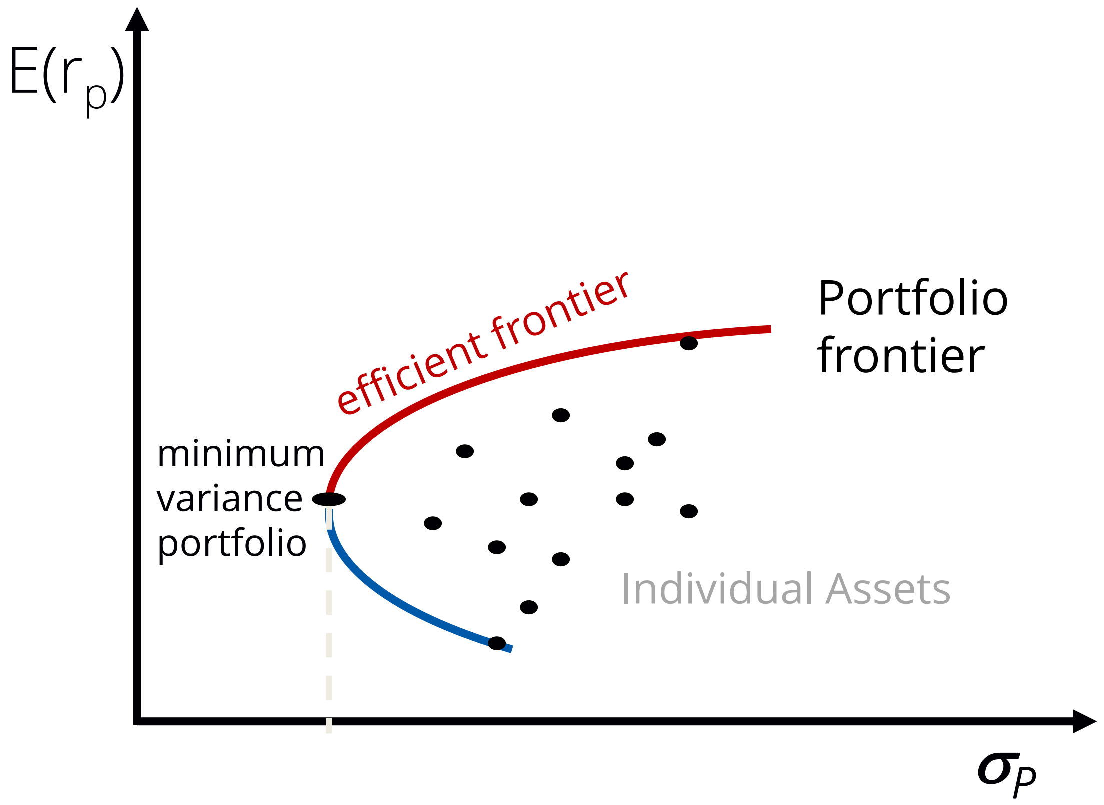
1. Find Portfolios of N assets that minimize volatility for a given level of expected return.
2. Do step 1 for all levels of expected return

Compared to the 2-assets case, curve moves north-west:

- Same $E(r_P)$ for lower σ_P^2
- Higher $E(r_P)$ for the same σ_P^2

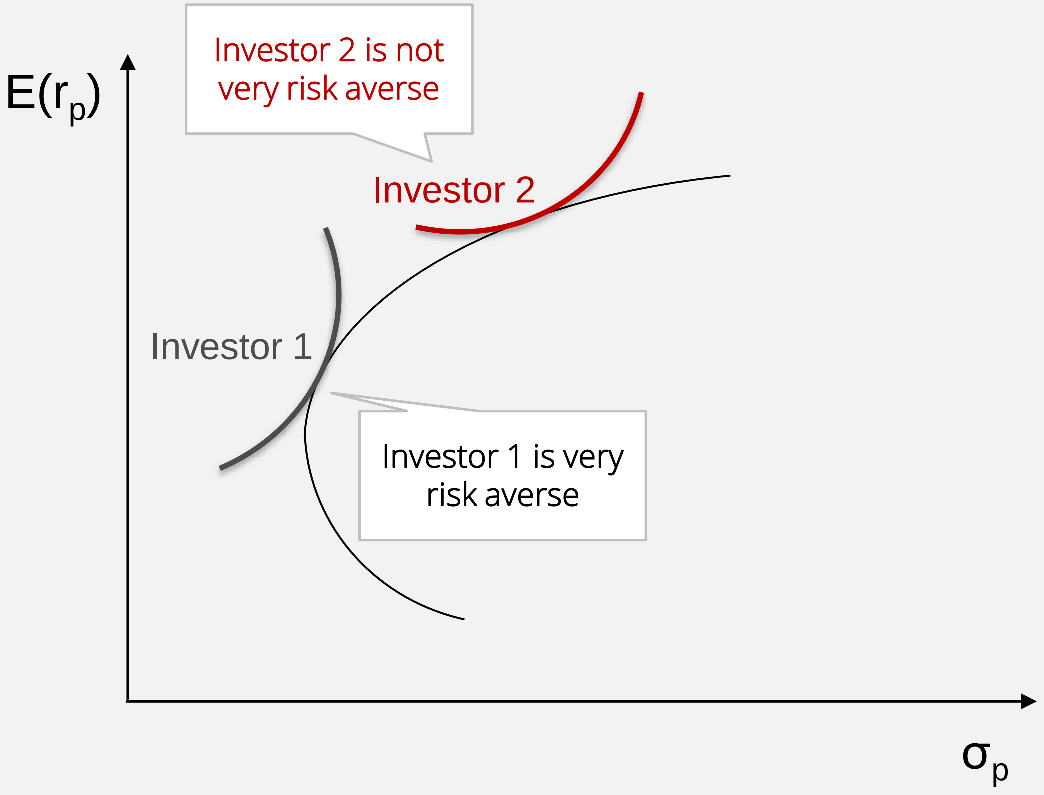
Efficient frontier for many securities

- The portfolio frontier is a hyperbola (once we allow unlimited short sales).
- Points to the right of the portfolio frontier are portfolios that can be formed, but which are not minimum variance. Such portfolios have high risk for their respective levels of expected return.
- Often, you will not find an individual asset on the portfolio frontier!



Optimal portfolio

Investor chooses the point where his/her indifference curve is tangent to the efficient set (as at this point (s)he's getting the highest possible utility).



Efficient portfolio of risky stocks and one risk-free asset

Introducing the risk-free asset

Properties

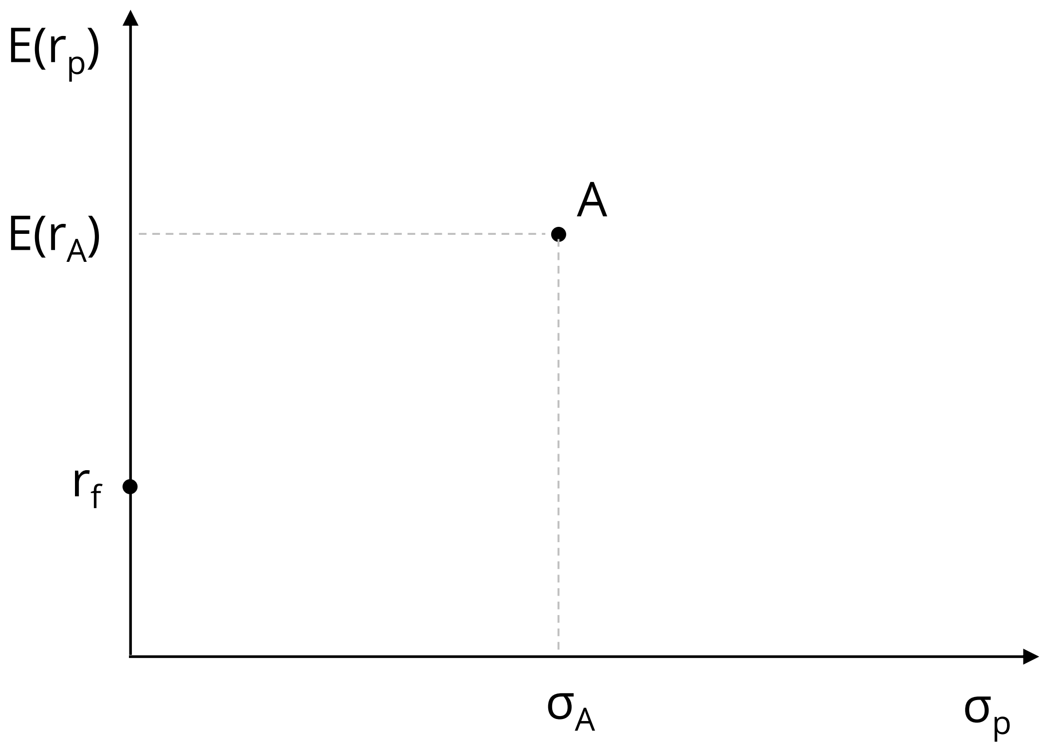
- Its return is known (in advance) to be r_f .
- The variance of the return on the risk-free asset is 0.
- The covariance of the return on the risk-free asset with the return on any other asset is 0.

Two possible implications

Reduce risk (and also $E(r_P)$) by investing in the risk-free asset, like T-Bills.

Increase risk (and also $E(r_P)$) by borrowing and investing even more in stocks.

Introducing the risk-free asset



What is the feasible set of the portfolios obtained from combining the r_f asset with a risky portfolio, A?

$$\sigma_p^2 = w_A^2\sigma_A^2 + w_f^2\underbrace{\sigma_f^2}_{=0} + 2w_A w_f \sigma_A \underbrace{\sigma_f \rho_{A,f}}_{=0}$$

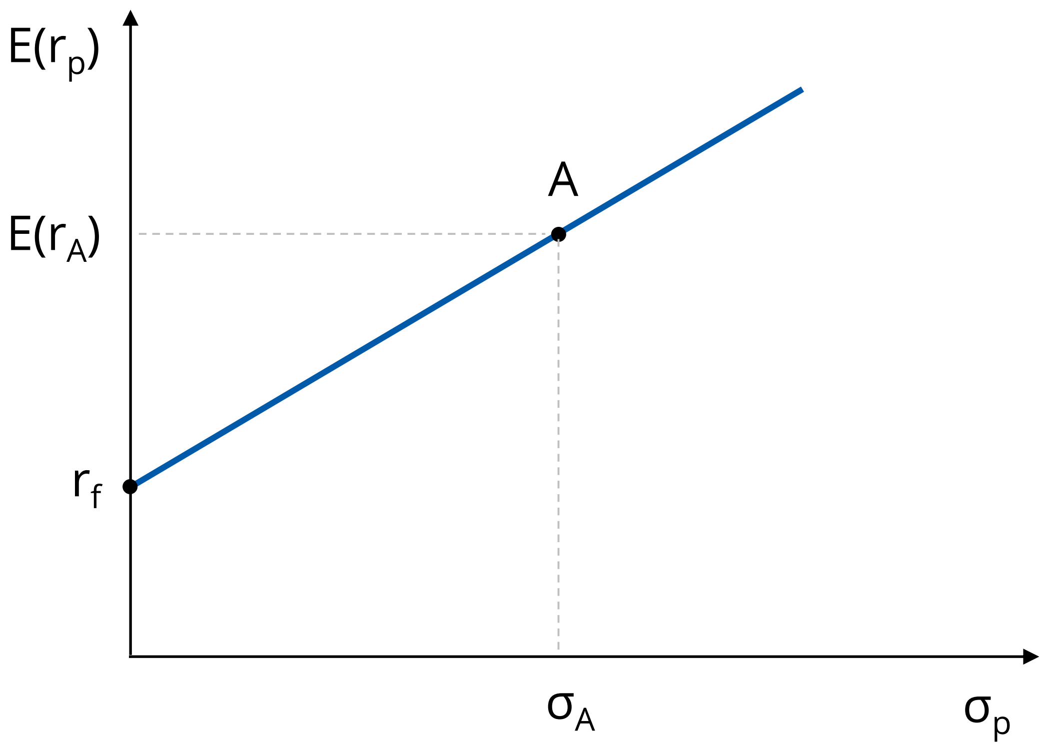
$$\sigma_p^2 = w_A^2\sigma_A^2 \leftrightarrow \boldsymbol{\sigma_p = w_A\sigma_A}$$

$$w_A = \frac{\sigma_p}{\sigma_A}$$

Result

$\sigma_P = w_A.\sigma_A$: volatility is a fraction (*equal to the share of wealth invested in the risky portfolio*) of the volatility of the risky portfolio.

Introducing the risk-free asset



What is the feasible set of the portfolios obtained from combining the r_f asset with a risky portfolio, A?

$$E(r_p) = w_A E(r_A) + w_f r_f$$

$$E(r_p) = w_A E(r_A) + (1 - w_A) r_f$$

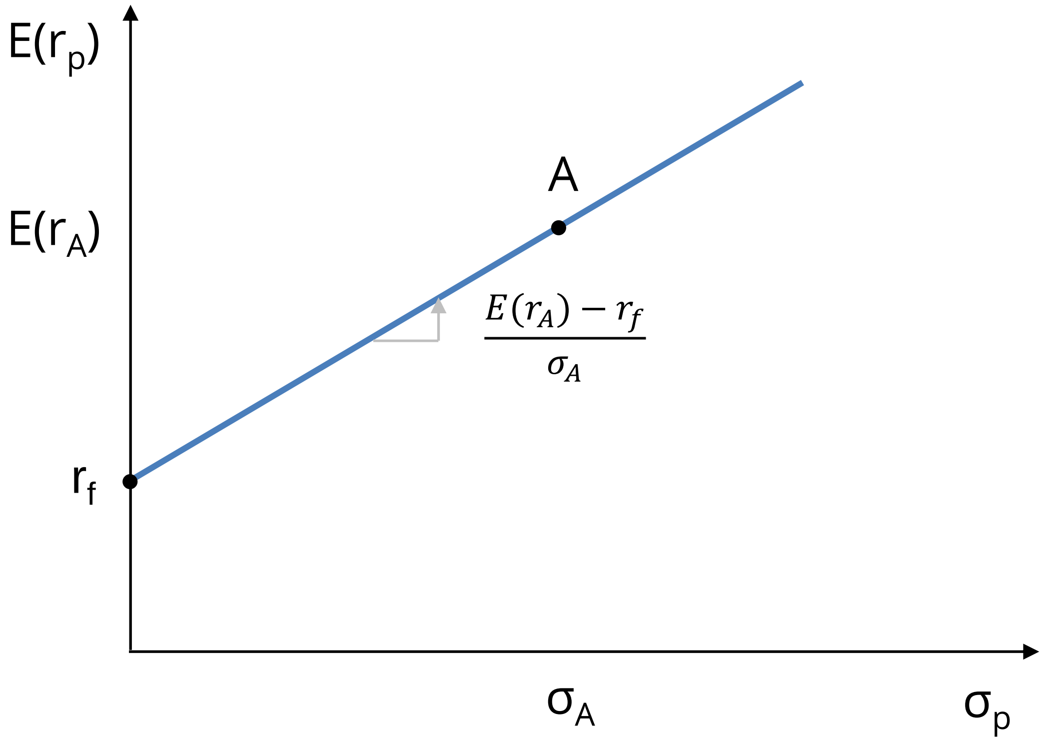
$$E(r_p) = \frac{\sigma_p}{\sigma_A} E(r_A) + \left(1 - \frac{\sigma_p}{\sigma_A}\right) r_f$$

Result

$$E(r_p) = r_f + \underbrace{\frac{E(r_A) - r_f}{\sigma_A}}_{\text{constant}} \sigma_p$$

→ Linear relationship

Sharpe ratio



$$E(r_p) = r_f + \frac{E(r_A) - r_f}{\sigma_A}\sigma_p$$

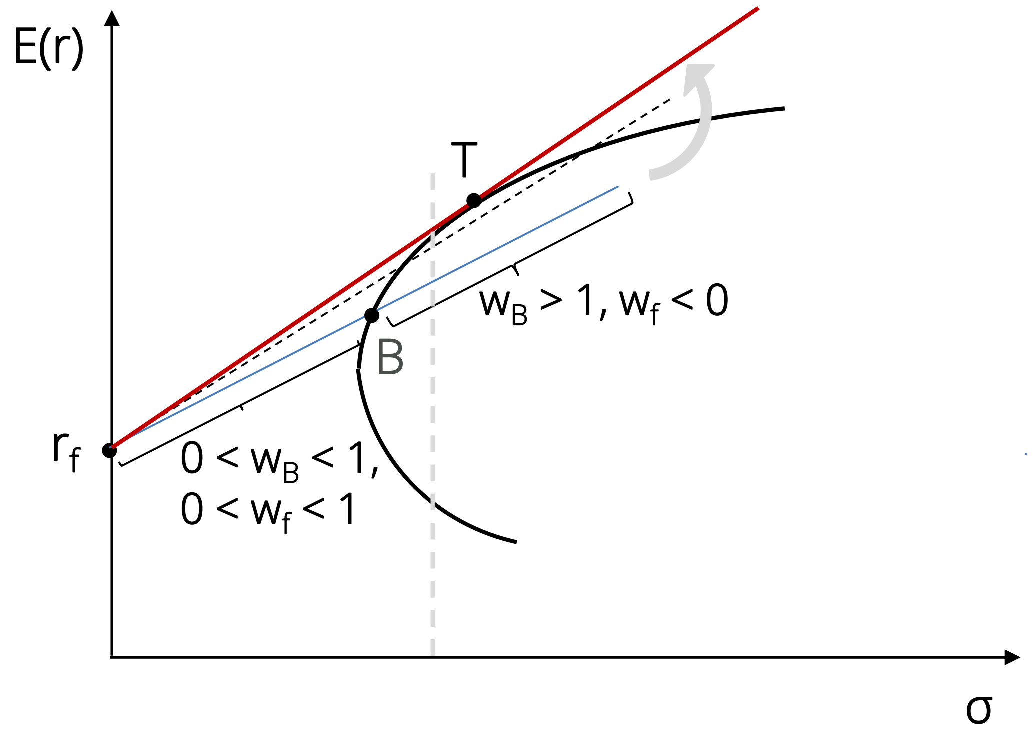
The slope of the line, $\frac{E(r_A)-r_f}{\sigma_A}$, is called the Sharpe ratio:

- Excess return per unit of risk

A investor wants to maximize the Sharpe ratio.

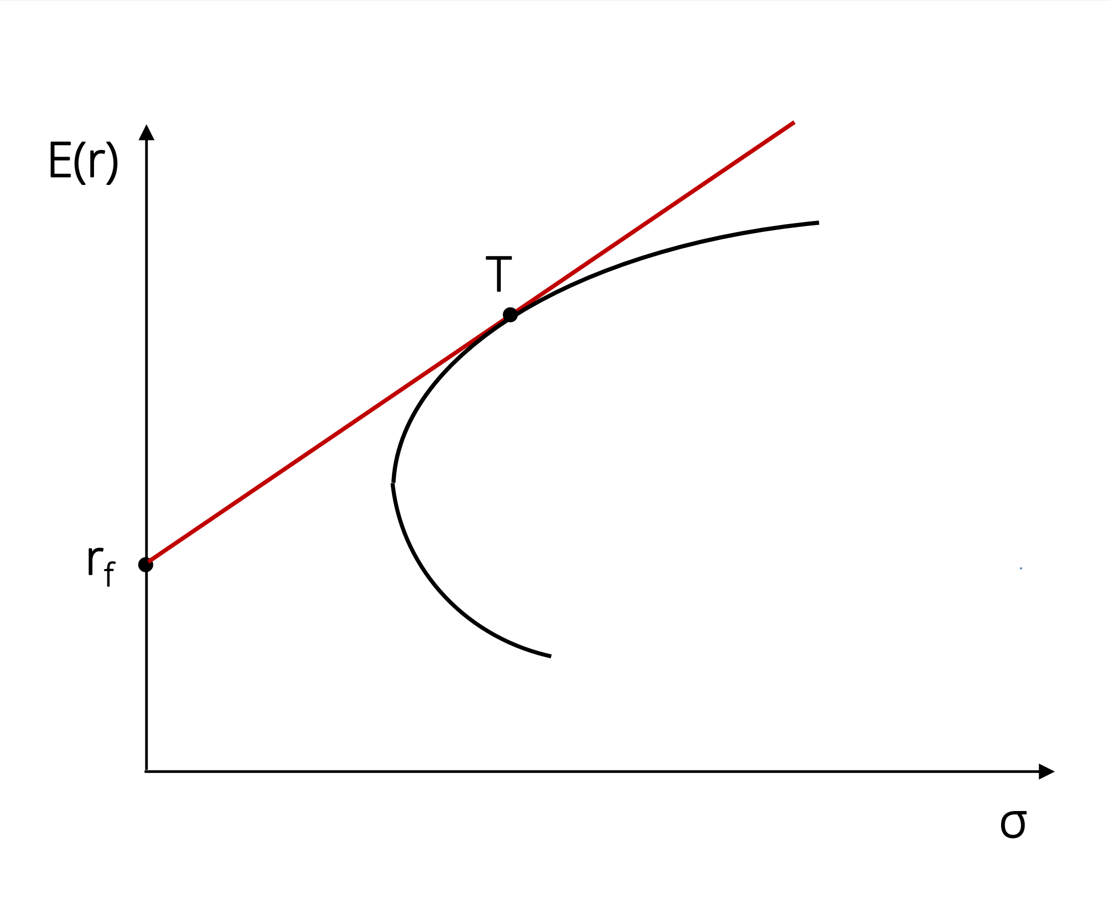
Which risky portfolio would you choose?

- **Question:** given that the set of portfolios generated by combining the risk-free asset and any portfolio is a straight line, what is the optimal portfolio of risky assets to select?
- If you combine the risk free asset with risky portfolio B on the efficient frontier, you can obtain return risk relation given by the blue line.
- You can do better by combining a risky portfolio in the portfolio frontier to the right of B. This will imply a larger Sharpe ratio.



Which risky portfolio would you choose?

- **Tangency portfolio** provides the highest possible compensation for every unit of standard deviation.
- When we include a risk-free asset, the straight line joining the tangency portfolio to r_f is the efficient set.
- All portfolios on this line have the same (maximum possible) Sharpe ratio.



Efficient frontier with risk free asset

All investors choose the same portfolio of risky assets – the tangency portfolio (T).

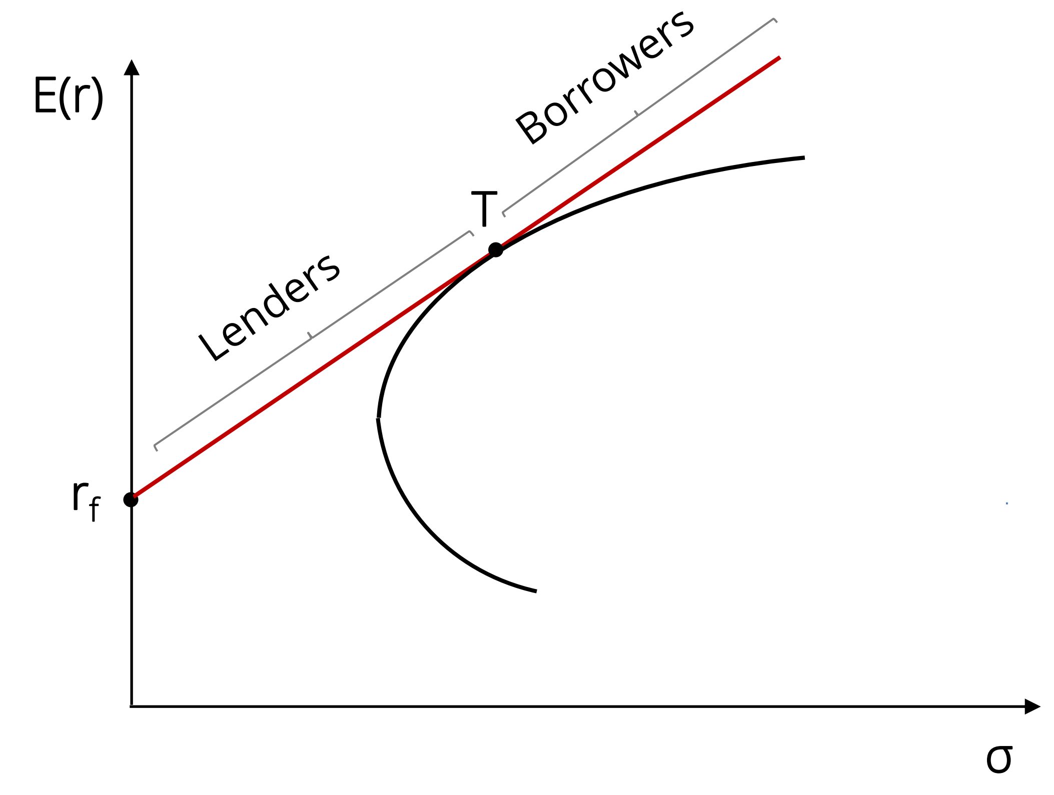
They then satisfy their personal preferences for risk by combining T with the risk-free security:

Lenders

Those who are very risk averse place a large amount of their wealth in the risk-free asset and only a little in T.

Borrowers

Those who are only slightly risk averse may choose to **borrow** money at the risk-free rate (i.e. short the risk-free asset) and invest more than their wealth in portfolio T.



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Capital Asset Pricing Model (CAPM)

CAPM

Assumptions

- There are N risky assets and a risk-free asset (N+1 assets in total). All investors can borrow/lend at the same risk free rate.
- Trading of assets is costless (no transaction costs or taxes).
- Investors care only about mean and variance (investors are rational mean variance optimisers).
- Investors have the same information (beliefs/homogeneous expectations).

All investors agree on the shape of the frontier of risky portfolios and agree on the exact composition of the tangency portfolio, T.

CAPM in equilibrium

Equilibrium in the markets for risky assets imposes the condition that demand for risky assets must be equal to the supply of risky assets.

Demand side

All investors hold risky assets in the proportions dictated by the tangency portfolio – thus, in portfolio terms, the **tangency portfolio (T)** summarises the demand for risk assets.

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Supply side

The stocks issued by all of the N firms in our world make up the supply of risky securities. The **market portfolio** summarises this supply of risky securities where the market portfolio is the value weighted portfolio of our N risky assets.

CAPM

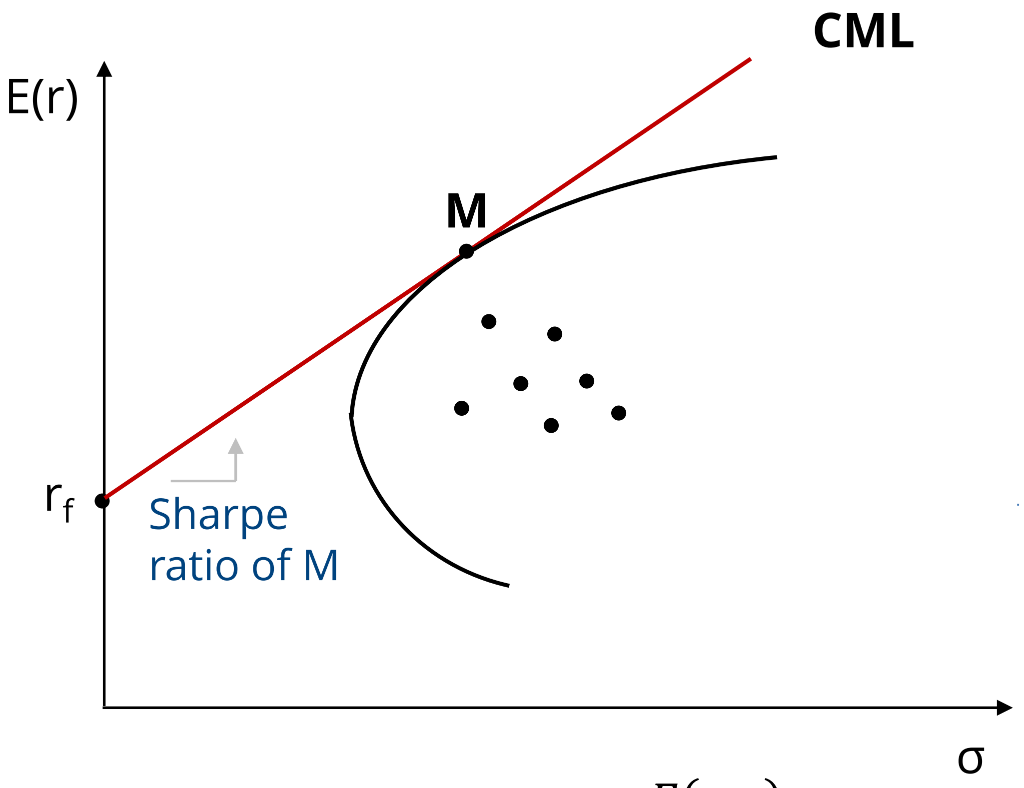
- In equilibrium, the tangency portfolio must be exactly the **market portfolio** of risky assets.
- All investors choose the same optimal portfolio of risky assets and this is the market portfolio.
- The weight of an asset in the market portfolio is its market capitalization divided by the total market cap of all assets.
- Every investor holds risky assets according to the market portfolio (M) weights and also holds some of the risk-free asset.

CAPM

There are two main relationships derived from the CAPM:

- Capital Market Line (CML)
- Security Market Line (SML)

CAPM - CML



$$\text{CML:} \quad E(r_i) = r_f + \underbrace{\frac{E(r_M) - r_f}{\sigma_M}}_{\text{M Sharpe ratio}} \sigma_i$$

- If CAPM holds i.e. in equilibrium, then efficient frontier is linear and consists of combinations of the risk-free asset and the market portfolio only. This is the Capital Market Line (CML).
- All investors locate somewhere on the CML, but all risky portfolios or securities other than the market portfolio lie to its right.
- The slope of the CML is the Sharpe ratio of the market portfolio.
- All of the portfolios on the CML have the same Sharpe ratio as that of the market portfolio.

CAPM – SML

Security Market Line

The CAPM equation - in this setting the relationship between risk and return is as follows:

$$E(r_i) = r_f + \beta_i (E(r_M) - r_f)$$

where $\boldsymbol{\beta_i = \frac{Cov(r_i, r_M)}{Var(r_M)}}$

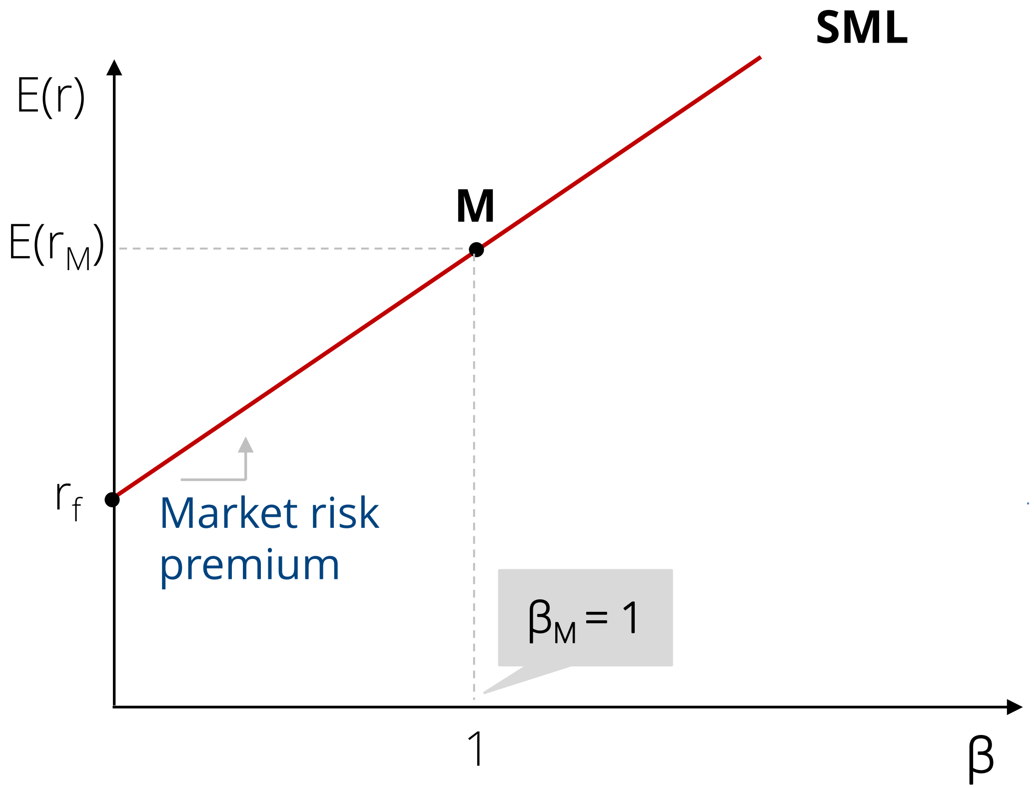
This is also known as the **pricing equation**.

$\boldsymbol{\beta_i}$

β_i is the sensitivity of a stock's expected return to the market return, in particular it measures the systematic risk:

β > 1	Asset is said to be **aggressive**: it tends to move with the market but the amplitude of its moves are greater than the market moves.
β < 1	Asset is said to be **defensive**: it tends to move with the market but its moves are smaller than the market moves.
β = 0	Asset carries no market risk and is said to be counter-cyclical.

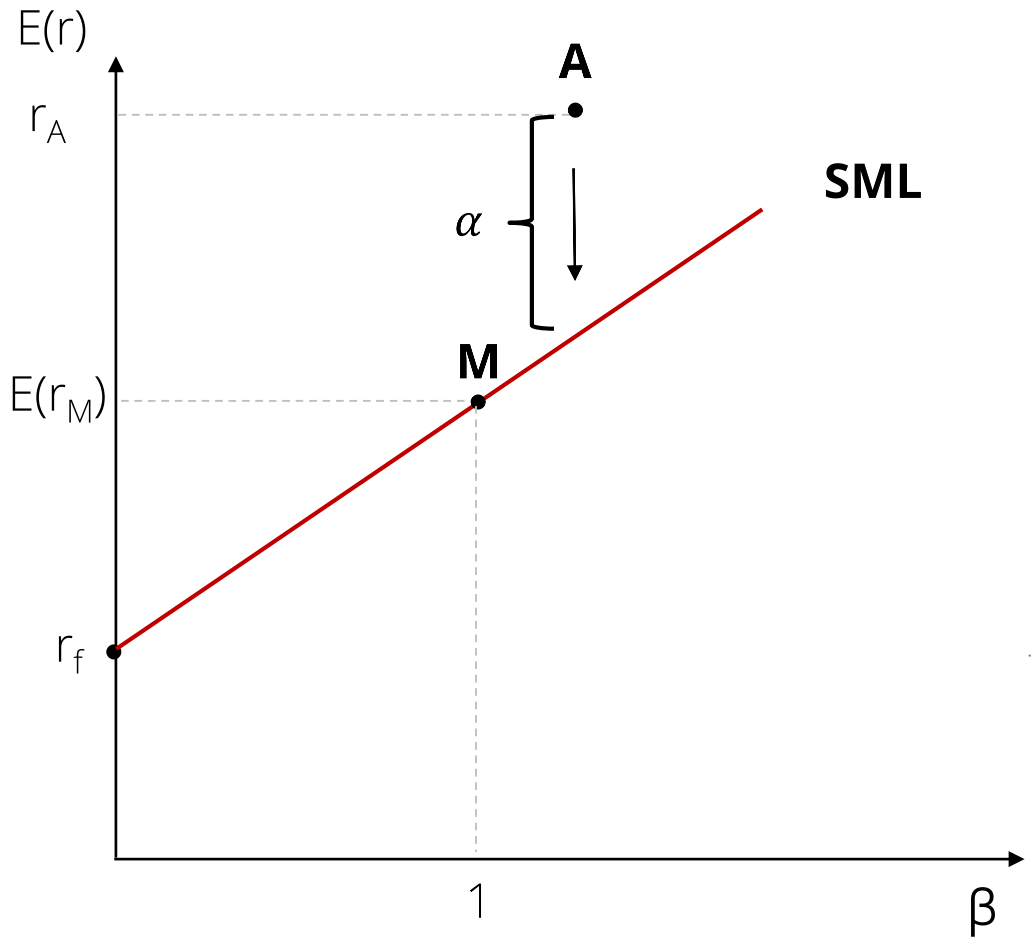
CAPM - SML



SML: $E(r_i) = \underbrace{r_f + \beta_i[E(r_M) - r_f]}_{\textbf{Required Return}}$

- Expected return is linearly increasing in systematic risk.
- A zero risk asset earns the risk-free rate (r_f).
- Market risk premium, or the price of risk, is $E(r_M) - r_f$.
- In equilibrium, <u>all portfolios or securities lie on the SML</u>. The only reason why one stock's expected return should differ from that of another, is if their βs are different.

Alpha



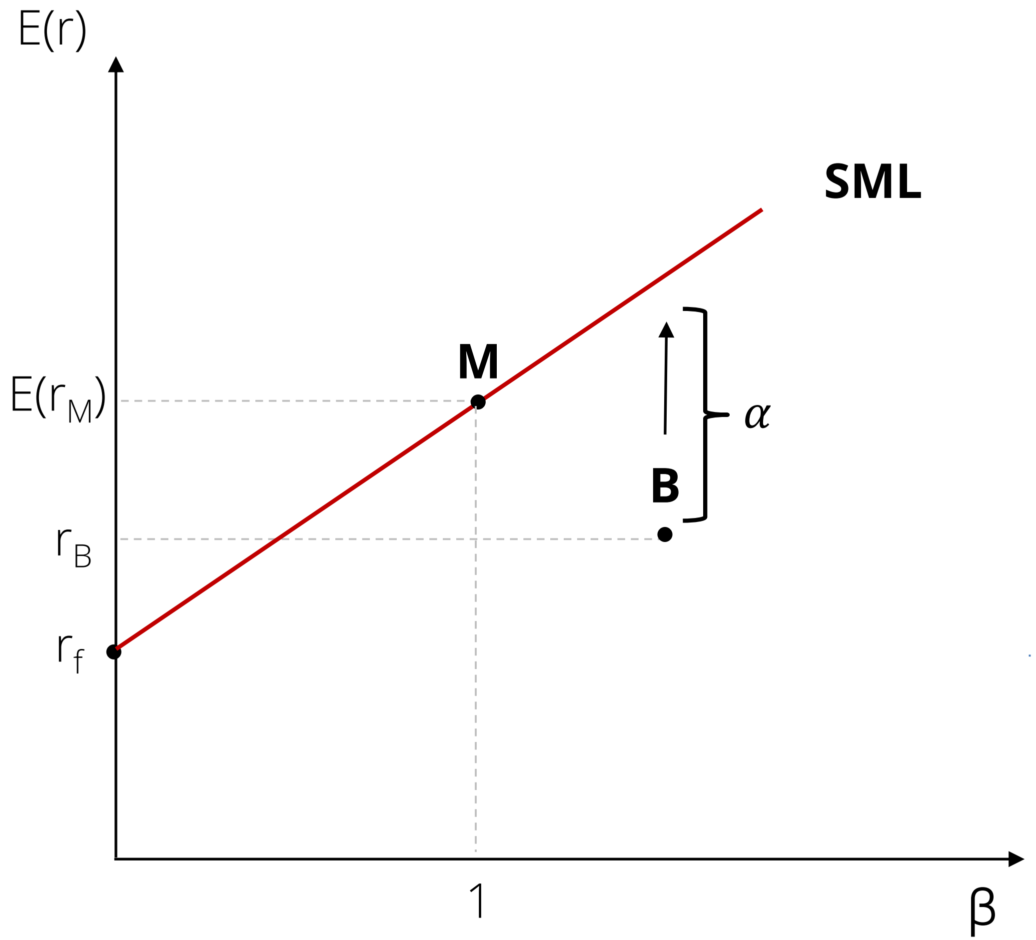
The difference between the observed return and the required return is called alpha:

$$\alpha_i = r_i - \underbrace{[r_f + \beta_i(E(r_M) - r_f)]}_{E(r_i)}$$

- If $\alpha > 0$, then the stock is generating an *abnormal* or *excess positive return* or is *underpriced*.
  - Adding more of this stock to the portfolio would increase Sharpe ratio

Demand will increase → P_i increases → r_i will decrease

Alpha



- If α < 0, then the stock is generating an *excess negative return* or is *overpriced*.
  - Adding more of this stock to the portfolio would decrease Sharpe ratio

Demand will decrease → P_i decreases → r_i will increase

Following the adjustments, the market portfolio is efficient again.

In a normal financial market, such pricing errors are taken care of by arbitrageurs quickly. ⇒ There exists no asset with an expected return higher (or lower) than what is implied by its systematic risk.

Beta of a portfolio

Since the SML applies to all tradable investments, it applies to portfolios as well. What is the β_p?

$$\beta_p = \frac{Cov(r_p, r_M)}{Var(r_M)} = \frac{Cov(\sum w_i r_i, r_M)}{Var(r_M)} = \sum w_i \underbrace{\frac{Cov(r_i, r_M)}{Var(r_M)}}_{\beta_i}$$

$$\boxed{\beta_p = \sum w_i \cdot \beta_i}$$

Beta of a portfolio is the weighted average of the securities in the portfolio.

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Application of the CAPM

Empirical evaluation of the CAPM

Consider the following version of the model:

$$E(r_i) - r_f = \alpha_i + \beta_i(E(r_M) - r_f)$$

Many of the results derived from data are broadly supportive of the CAPM , but some very important rejections of these implications exist:

- The SML we see in the data seems to have a lower slope than the one predicted by the model.
- Factors other than β explain expected returns:
  - Size: small stocks tend to have higher expected return than big stocks (holding β constant)
  - Value: mean returns on stocks with high book-to-market tend to be larger than those on low book-to-market stocks (holding β equal)

How to apply the CAPM?

Questions to be answered:

- What is the risk-free asset?

- What is the market?
  - What is the expected return of the market?

- How to estimate the beta?

CAPM inputs: risk-free rate

What risk-free rate should be used?

- In theory: short-term (1 month) treasury-bill rate.
- In practice, most investors pay a substantially higher rate to borrow funds:
  - Because they borrow funds for a longer period!
- Surveys suggest most practitioners use 10 to 30 year treasury bond yield as input to CAPM.

CAPM inputs: Market portfolio and beta

Market portfolio

- CAPM says it should be all the assets in the world ⇒ unobservable!
- Typically people use *broad*, *value-weighted* US stock market index: a portfolio whose return likely closely matches the return of the true market portfolio (all NYSE, AMEX, and NASDAQ firms).

Estimating Beta

How to estimate beta?

- The standard procedure for estimating betas is to regress excess stock returns against excess market returns:

$$\underbrace{E(r_i) - r_f}_{y_i} = \alpha_i + \beta_i \underbrace{(E(r_M) - r_f)}_{x_m}$$

  - $y_i = a + b * x_m$
  - where a is the intercept and b is the slope of the regression.
- The slope of the regression corresponds to the beta of the stock.

Key takeaways

01 **Understand mean-variance optimization with risky stocks and a risk-free asset.**

02 **Capital Asset Pricing Model and intuition of its derivation.**

03 **Understand the differences between SML and CML.**