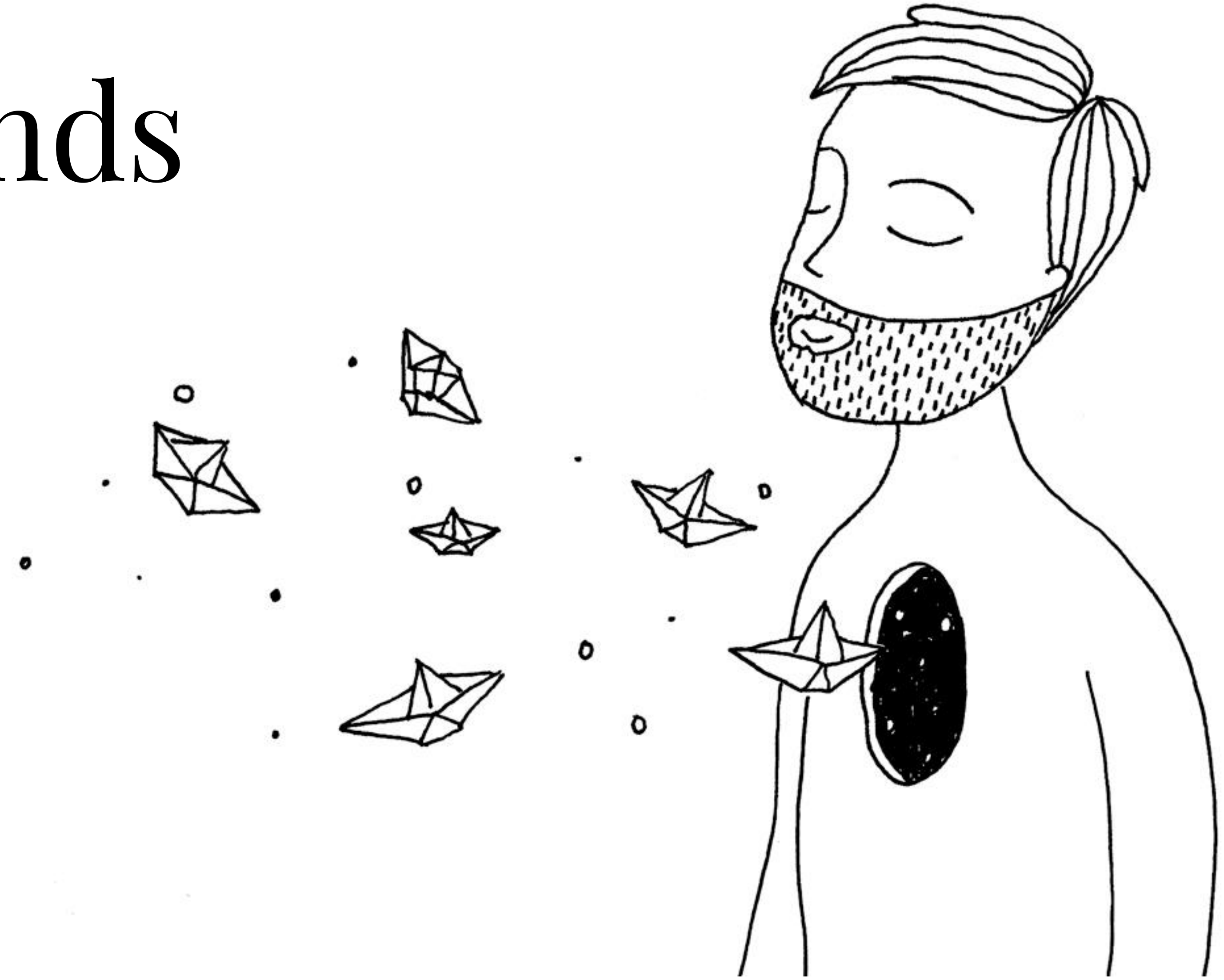
Stocks and Bonds



Advanced Financial Management

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Overview

Stocks:

- How to value stocks
- Organic Growth

Bonds:

- How to value bonds
- Duration

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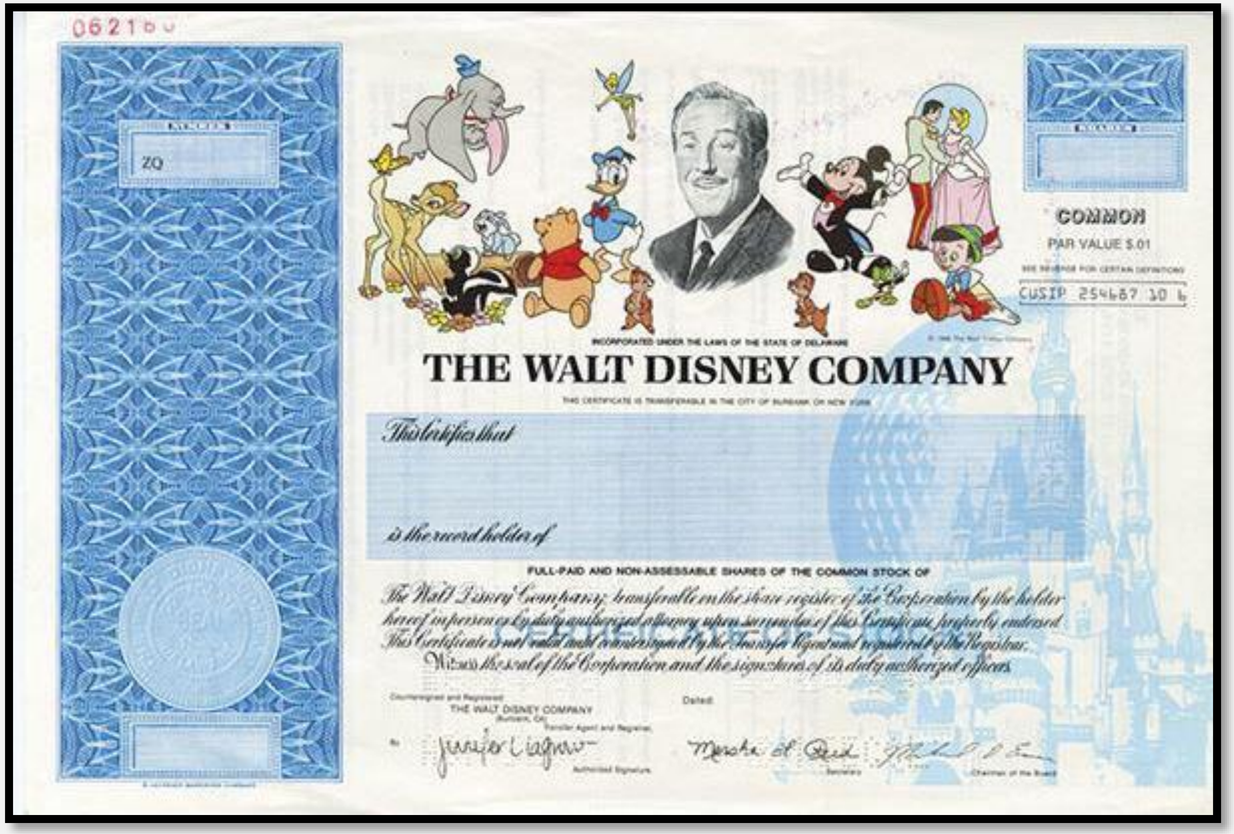
Stocks – How to value stocks?

Common Stock

- Common Stock (or Equity) is a security representing a share in the ownership of a corporation.
- Stockholders are the owners of the firm.
  - They have the right to vote on company policy and strategy, whereas bond holders do not.
  - They are entitled to the residual cashflows from the firm's operations.



Common stock - terminology

Terms	Description
Initial Public Offering	The first sale of stock in a corporation to the public.
Secondary Market	A market, often a stock exchange, in which previously issued shares are traded amongst investors.
Dividends	Payments made by companies to shareholders. These are usually ex-ante uncertain (unlike bond coupons).
Dividend yield	Ratio of annual dividend to share price.
Price ex-dividend	Price at which the stock is trading right after the current dividend has been paid.

Common stock - terminology

Terms	Description
Market Value	The total stock market value of the firm's stock (i.e. price per share multiplied by number of shares outstanding).
Book Value	Accounting value of the firm's equity as reflected on the company's balance sheet.
Liquidation Value	The amount that would be available to shareholders if the firm was liquidated and all creditors paid off.

How to value a stock

General principal of valuation

Computation

The value of a security is the PV of expected future cash flows.

$$P = PV(All\ future\ CFs)$$

This is equivalent to:

$$NPV\ (buying\ fin.assets) = -\underbrace{Inv}_{P} + PV\underbrace{(benefits)}_{\text{All future CFs}}$$

$$NPV(buying\ fin.assets) = 0$$

What are stocks' cashflows?

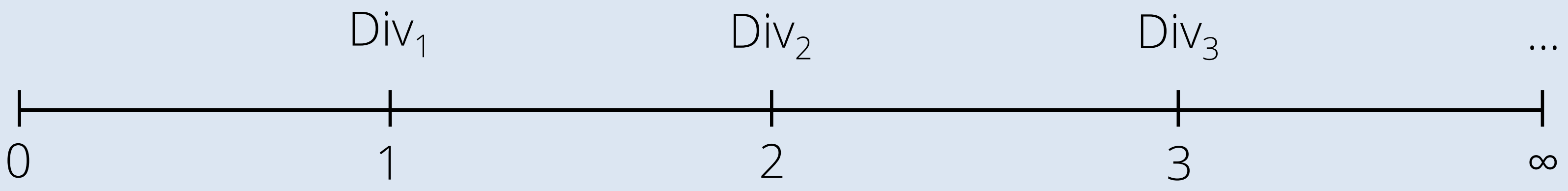
Stocks pay **dividends**. There is uncertainty about these CFs:

1. *Size* and *Timing*
2. What is the appropriate discount rate

   The rate should be appropriate to the risk presented by the stock (instead of using the treasury yield curve)

Dividend discount models

Computation



$$P = \frac{Div_1}{1+r} + \frac{Div_2}{(1+r)^2} + \frac{Div_3}{(1+r)^3} + \cdots$$

Questions

1. Where are capital gains incorporated?
2. How can we determine all future dividends?
3. What is the relevant discount r?

Dividend discount models

1. The dividend discount model (DDM) does not ignore capital gains even though they may not appear explicitly in the formula

2. How do we forecast future dividends?
   - Constant dividends
   - Constant growth rate. How to estimate this? More elaborate models

3. What discount rate should be used?
   - The expected return that can be obtained in the stock market for investments of similar risk.
   - This is the *opportunity cost* for an investor who wants to buy this stock.

How do DDM incorporate Capital Gains?

Hold a stock for one year

0 — 1

Dividends in the year you hold the stock: Div_1

Value of selling: P_1

Computation

$$P_0 = PV(future\ CFs)$$

$$P_0 = \frac{E_0[D_1 + P_1]}{(1+r)}$$

Hold a stock for another year

0 — 1 — 2

P_1 — Div_2, P_2

Holding between year 1 and 2

$$P_1 = \frac{E_1[D_2 + P_2]}{(1+r)}$$

Computation

$$P_0 = \frac{E_0\left[D_1 + \frac{E_1[D_2+P_2]}{(1+r)}\right]}{(1+r)} = \frac{E_0[D_1]}{(1+r)} + \frac{E_0[D_2]}{(1+r)^2} + \frac{E_0[P_2]}{(1+r)^2}$$

How do DDM incorporate Capital Gains?

If we continue to reinvest/hold the stock forever then

$$P_0 = \frac{E_0[D_1]}{(1+r)} + \frac{E_0[D_2]}{(1+r)^2} + \frac{E_0[D_3]}{(1+r)^3} + \cdots$$

We are assuming constant required return: same *r* in the future

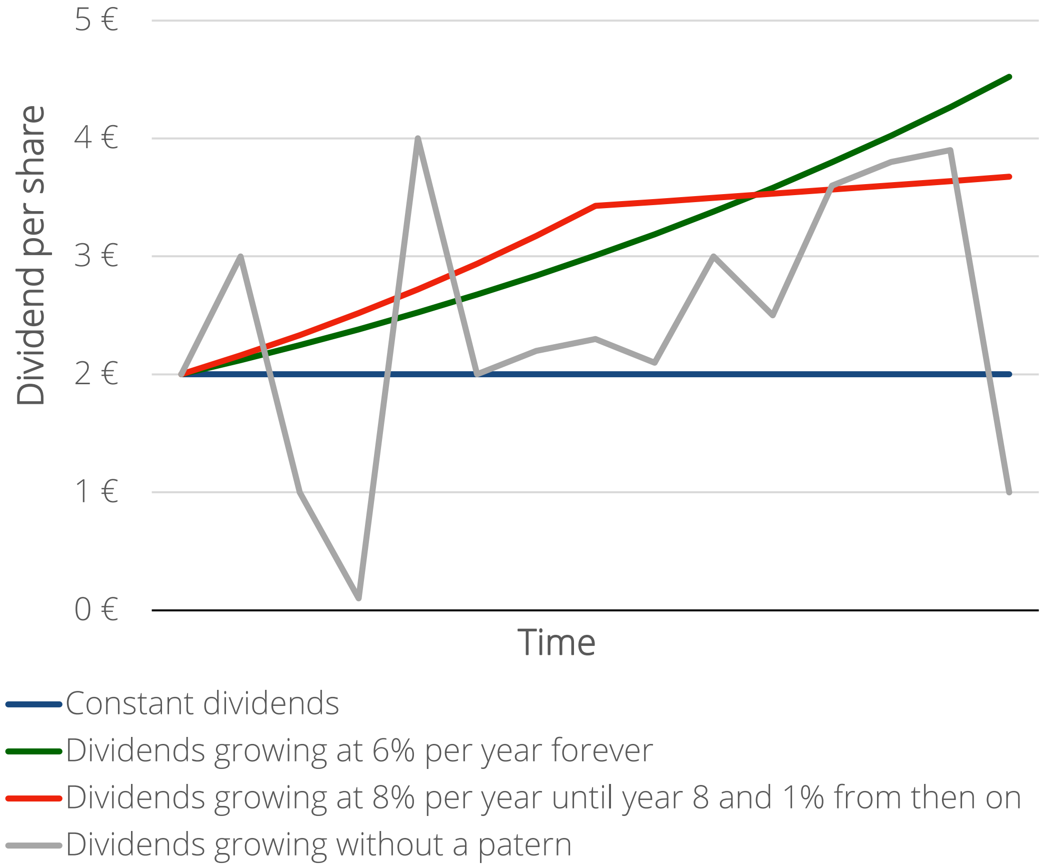
- This is the same formula for the price of a stock we had before (except now we are taking account the uncertainty about what the dividend is).

Key takeaways:

- Prices will be greater when expected dividends are greater
- Prices will be lower when the expected return required by investors (r) rises
- Prices take implicitly the capital gains into account

Estimating dividends: Dividend Growth Model

Dividend scenarios



Dividend scenarios

Constant Dividends	Perpetuity
Dividends with constant growth	Growing perpetuity
Dividends with differential growth	Annuity + Perpetuity
Dividends without a pattern	Need to discount individually

Constant dividends

- If we expect all future dividends to be constant at a level of D. Then the price becomes

$$P_0 = \sum_{n=1}^{\infty} \frac{D}{(1+r)^n}$$

- This is just a perpetuity and so the PV is

$$P_0 = \frac{D_1}{r}$$

Example

A company is going to pay €3 in dividends per share every single year and forever, starting one year from now. Shareholders required rate of return is 10%. (a) What is the current share price? (b) What is the weight of long-term versus short-term in the share price?

$$\text{Share Price} = S_0 = \frac{3}{10\%} = 30 \quad \text{(a)}$$

Horizon	Years	PV(CF)	Weight
Short-term	1 to 3	7.46	24.9%
Long-term	4+	22.54	75.1%
		30.00	100.0%

(b)

Even though the long-term has fundamentally a larger weight in the share price, some managers might focus on the short-term because (1) short-term quarterly earnings provide information to the market and (2) their bonuses are tied to short-term earnings. This can be mitigated by building compensation plans with stocks and stock options to incentivize long-term growth and profitability.

Gordon Growth model

- Is it reasonable to assume that the *dividend* is constant?
  - Dividends are sticky (firms are particularly reluctant to cut), but…
  - … dividends follow earnings:
    - For mature companies it is reasonable to assume earnings (and therefore dividends) grow at a constant future rate (g)
- Denote next period's expected dividend by D and the growth rate by g, so that the stream of expected dividends will be D, D(1 + g), D(1 + g)^2 …
  - This is just a perpetuity with growth and its PV is

$$P_0 = \frac{D_1}{r - g}$$

Gordon Growth model

- Assumes dividends grow at a constant rate.
- Implies:
  - stock prices are higher when dividends or their growth rates are higher
  - stock prices fall when required returns rise
  - To maximize share price the firm faces a trade-off: increasing dividends usually means lower investments and therefore lower future growth:
    - cutting dividends to invest increases the share price if and only if the new investments have a positive NPV

$$\boldsymbol{P_0 = \frac{D}{r-g}}$$

Example

What is the value of a stock that is expected to pay a $3.00 dividend next year, and then increase the dividend at a rate of 8% per year, indefinitely? Assume a 12% expected return.

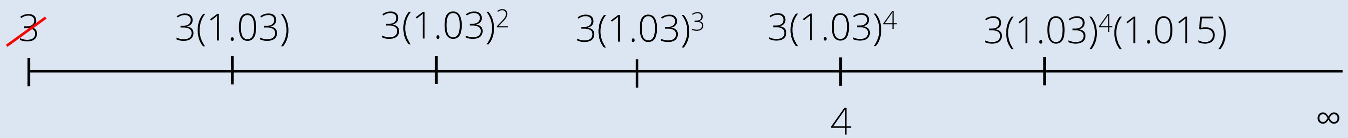
$$P = \frac{3}{0.12 - 0.08} = 75$$

Multiple stages of growth

Firms May Have Multiple Stages of Growth.

- Growth Stage: rapidly expanding sales, high profit margins, and abnormally high growth in earnings per share, many new investment opportunities, low dividend payout ratio.
- Transition Stage: growth rate and profit margin reduced by competition, fewer new investment opportunities, high payout ratio.
- Mature Stage: earnings growth, payout ratio and average return on equity stabilizes for the remaining life of the firm.

Example



A firm just paid its annual dividend of €3. The dividends are expected to grow from now on at 3% until year 4 and from then on it will grow at 1.5% per year forever. If r=10%, what is the current share price?

$$P = \frac{3(1.03)}{0.1 - 0.03}\left(1 - \left(\frac{1.03}{1.1}\right)^4\right) + \frac{3(1.03)^4(1.015)}{0.1 - 0.015}\frac{1}{1.1^4} = 37.75$$

Expected returns to holding a share

- Recall the value of a stock can be seen the value of holding a stock for one year:

$$P_0 = \frac{E_0[D_1 + P_1]}{(1+r)}$$

- We can rewrite to get the percentage one year return from holding the stock is:

$$r = \frac{E_0\,[D_1 + P_1 - P_0]}{P_0} =$$

$$= \underbrace{\frac{E_0\,[D_1]}{P_0}}_{\text{Dividend yield}} + \underbrace{\frac{E_0\,[P_1 - P_0]}{P_0}}_{\text{Capital gain}}$$

  - Thus, the return on a stock is equal to the expected dividend yield plus the expected capital gain.

Stock growth rate

What is the stock price in 1 year?

P0

$$P_1 = \frac{D_2}{r-g} = \frac{D_1(1+g)}{r-g} = \boxed{\frac{D_1}{r-g}}.(1+g)$$

$$P_1 = P_0(1+g) \leftrightarrow$$

$$\frac{P_1}{P_0} - 1 = g \quad \leftrightarrow \quad \frac{P_1 - P_0}{P_0} = g$$

Thus, g is also the growth rate of the stock price!

- Thus we can rewrite $r = \frac{D_1}{P_0} + g$

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Stocks – Organic Growth

Stock prices, earnings and dividends

Term	Notation	Description
Payout ratio	p	Share of firm's earnings paid as dividends.
Plowback ratio	b	Share of earnings retained by the firm and used for investment.
Book value of Equity	BV	Accounting value of equity.
Return on Equity	ROE	Measure of the amount of earnings that 1 Euro of equity (book value) creates
Growth rate	g	Growth rate of dividends, earnings and stock prices (under some conditions)
Net income	Net income	Earnings after paying interest and taxes is also known as the firm's net income
Earnings per share	EPS	Earnings or net income divided by the number of outstanding shares in the firm

Stock prices, earnings and dividends

Balance sheet at t=0

Assets 1000

Equity 1000

BV

Profit by the end of year 1

ROE 10% → Net income 100

- Assume 1 share in this firm.

Formulas and intuition

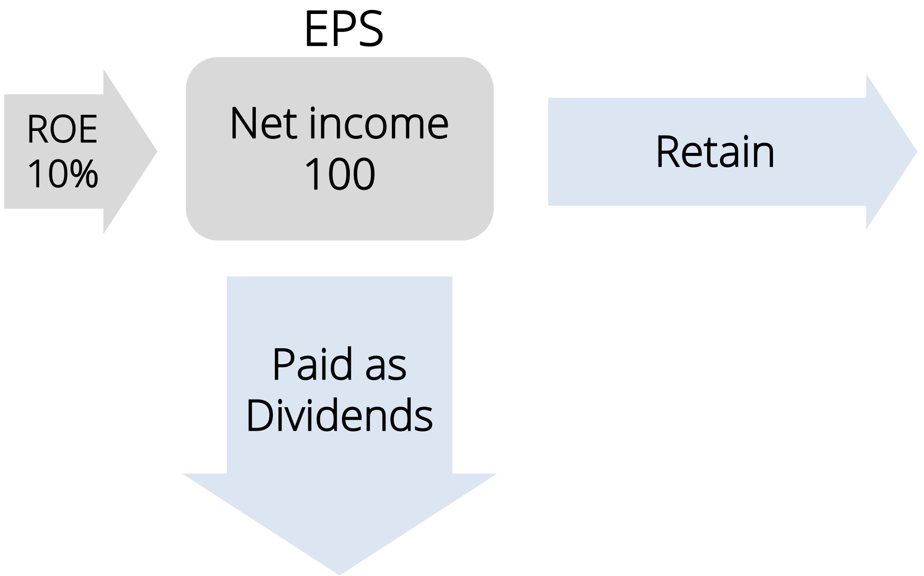
- ROE = 10% means 1 Euro of Equity generates €0.1 of Net Income:

$$Net\ income_t = ROE_t \times BV_{t-1}$$

$$\boldsymbol{ROE_t = \frac{Net\ income_t}{Book\ value\ of\ Equity_{t-1}} = \frac{EPS_t}{BV^{ps}_{t-1}}}$$

Stock prices, earnings and dividends

Profit by the end of year 1	What happens to income?	Formulas and intuition



$$Inv_t = b \times EPS_t = 0.2 \times 100$$

The amount retained is invested in the firm.

$$Div_t = p \times EPS_t = 0.8 \times 100$$

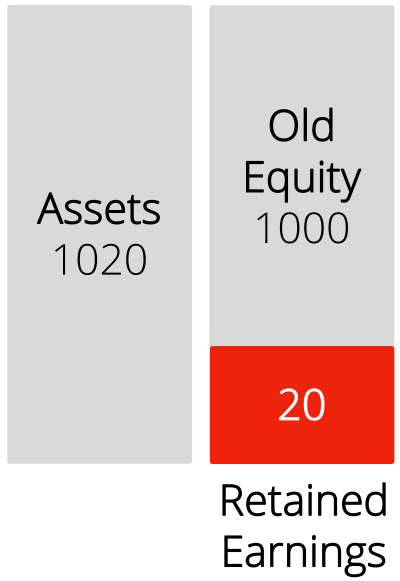
Shareholders receive fraction p of the earnings generated by the firm.

Note: Earnings are either paid out or reinvested so the payout ratio and plowback ratio need to add up to 1: $b + p = 1$

Stock prices, earnings and dividends

What happens to income?	Balance sheet at t=1	Formulas and intuition

Retain (b=20%)



- The Book Value of Equity at t=1 will incorporate the new investment, i.e., the retained earnings:

$$BV_t = BV_{t-1} + Investment_t$$

Stock prices, earnings and dividends

Retained earnings can be invested at firm's return on equity:

$$g = ROE.b = ROE.(1 - p)$$

NOTE: this assumes constant ROE and constant plowback ratio

Intuition:

- Plowback tells us how much of current earnings is retained for investment purposes and ...
- ROE tells us how much each Euro of equity contributes to earnings
- Put them together than you get growth in earnings (g)

Growth opportunities: practical example

Example

A company is generating €10 annual EPS and its discount rate is 18%. (a) What is its share price without retained earnings? (b) What is its share price assuming the company will retain and reinvest 60% of its earnings every year in projects that will generate a ROE of 25% per year?

	Without retention				With retention				
Year	EPS	Div (100%)	Reinvest-ment (0%)	NPV of Re-investment	EPS	Div (40%)	Reinvest-ment (60%)	Annual return of investment	NPV of Reinvestment
1	10	10	0	0	10 x (1+g) ↓	4 x (1+g) ↓	6 x (1+g) ↓	6 x 25%=1.5 x (1+g) ↓	$-6+\frac{1.5}{18\%}\approx 2.33$ x (1+g) ↓
2	10	10	0	0	10+1.5= 11.5 x (1+g) ↓	4.6 x (1+g) ↓	6.9 x (1+g) ↓	6.9 x 25%=1.725 x (1+g) ↓	$-6.9+\frac{1.725}{18\%}\approx 2.68$ x (1+g) ↓
3	10	10	0	0	11.5+1.725 =13.225 x (1+g) ↓	5.29 x (1+g) ↓	7.935 x (1+g) ↓	7.935 x 25%≈1.98 x (1+g) ↓	$-7.935+\frac{1,98}{18\%}\approx 3.08$ x (1+g) ↓
(...)									

g = ROE x b
= 25% x 60% = 15%

$S_0 =$ "Cash-cow" value $= \frac{10}{18\%} = 55.55$ (a)

$S_0 = \frac{4}{18\% - 15\%} = 133.33$ (b)

Present Value of Growth Opportunities - example

The difference in value between the firm that retains and reinvests and the firm that does not is called the Present Value of Growth Opportunities (PVGO).

- In the example the PVGO = €133.33 - €55.55 = €77.78 per share.

The PVGO is the discounted value of earnings obtained from reinvesting a fraction of previous year's earnings.

Example

Year 1: the NPV of reinvestment $= -6 + \frac{1.5}{0.18} = 2.33$

Year 2: the NPV of reinvestment = 2.68

$$PVGO = \frac{NPV_1}{r-g} = \frac{2.33}{0.18 - 0.15} = 77.78$$

Present Value of Growth Opportunities - example

Intuition:

- The firm that plows back earnings has a greater stock price.
  - This is because its ROE is greater than the required return (r) so retaining earnings generates returns that are above the discount rate.
- If the ROE was equal to the required return, the firm would be worth the same in the two scenarios:
  - Assume firm's ROE changes to 18%. Then g = 0.18 x 0.6 = 0.108
  - What happens to price?

$$P = \frac{4}{0.18 - 0.108} = 55.55$$

PV of growth opportunities

The stock price can be decomposed into two components

- Present value of earnings under a no-growth policy
- Present value of growth opportunities

$$P = P^{NG} + PVGO$$

$$PVGO = P - P^{NG}$$

Computation

P^{NG} is computed assuming no growth and thus no plowback:

- $b = 0 \Leftrightarrow p = 1$
- Since Div = p x EPS = EPS

This implies:

$$PVGO = P - \frac{EPS_1}{r}$$

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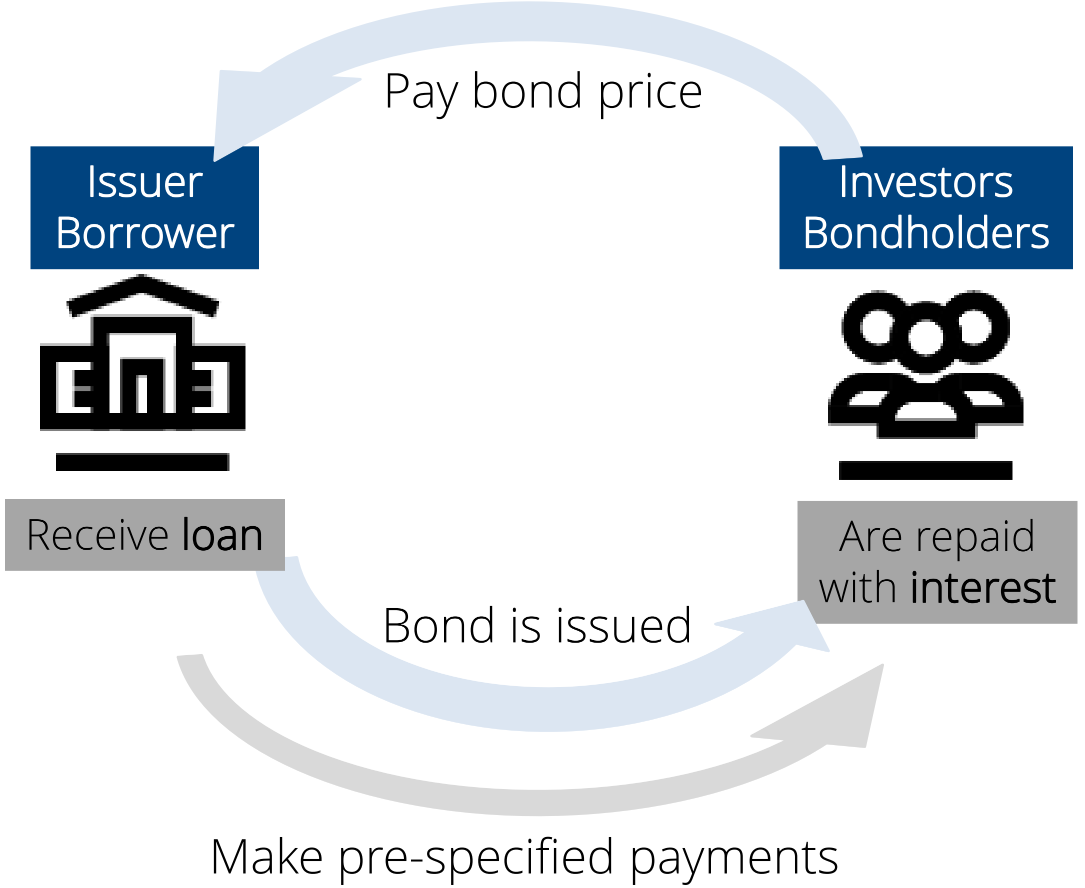
Bonds – How to value bonds?

Introduction

Terminology

- Bonds are often called fixed-income securities.
- They are so named as the cashflows they deliver to an investor, as well as the dates that these cashflows will arrive, tend to be known (fixed) in advance.

Basic structure



Terminology

Term	Notation	Description
Face Value	FV	Also known as par value, principal or redemption value. This is the value the investor receives from the issuer at maturity.
Maturity date	T	Final repayment date, that is, the date at which the FV is promised to be paid.
Coupon	C	Coupons are the interest payments made to investors promised by the issuer.
Coupon rate	c	Percentage of face value which is paid in Coupons.
Coupon frequency		Frequency coupons are paid to investors.

Different types of bonds

- Zero coupon bond / pure discount bonds
- Coupon bonds
- Consols

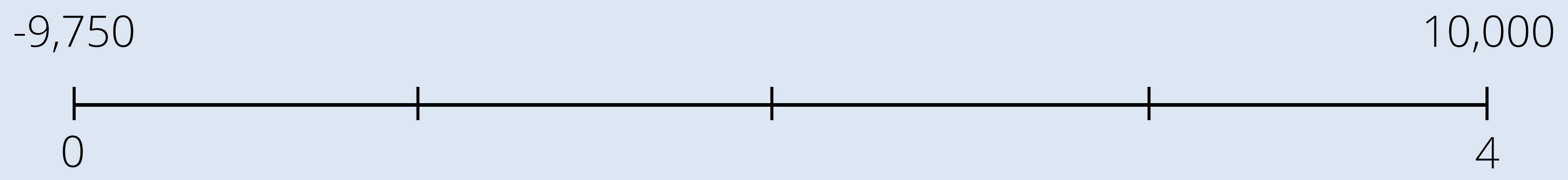
Zero coupon bond

T-year zero coupon bond

- Promises the purchaser a single payment (the face value) T years from the current date. The date of the payment is called the maturity date and T is the time to maturity.

Example

4-year zero with FV=€10,000 with a price of €9,750.



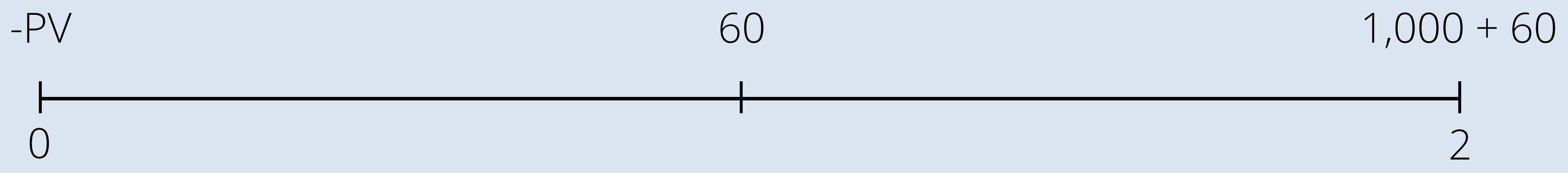
Coupon bonds

T-year coupon bond

- Bondholder receives a coupon payment at regular intervals until maturity. These payments could be made, annually, semi-annually or quarterly.
- These coupon payments are usually the same at every payment date.
- Coupon rate is expressed as an APR (annual proportional rate)
- At T, the bondholder receives both a coupon payment and the face value

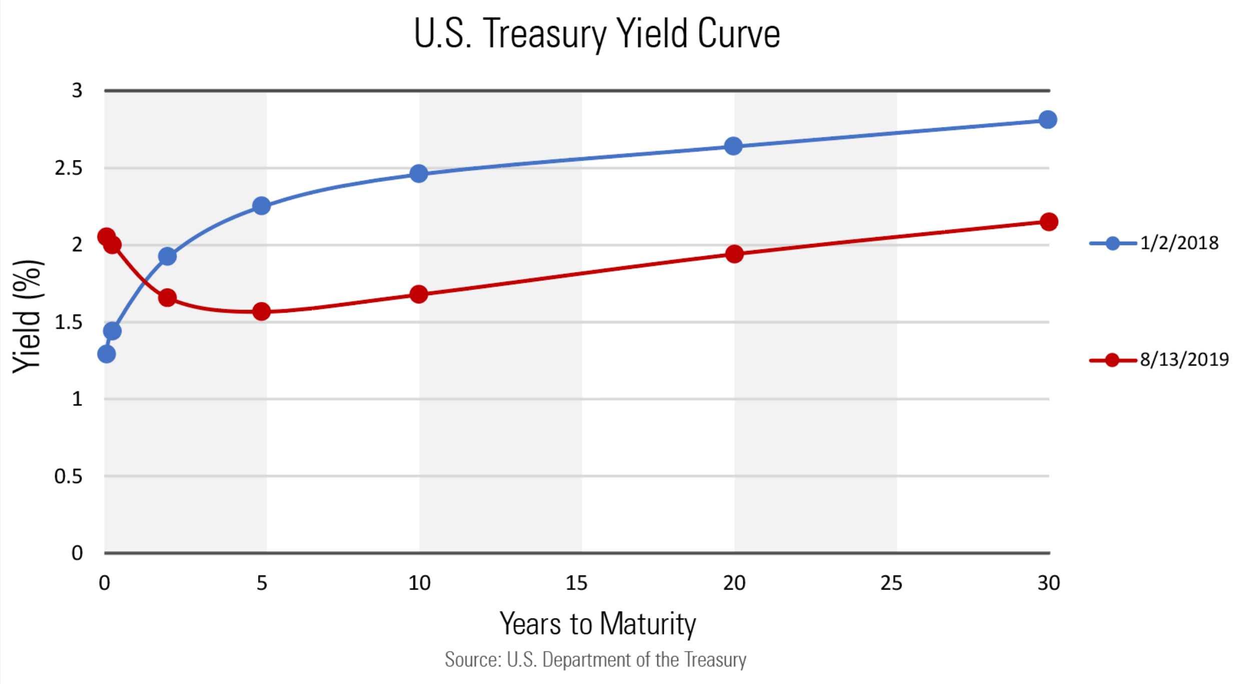
Example

2 year 6% coupon with annual coupons and face value of €1000.



Yield curve

- Financial markets quote different interest rates for different maturities.
- **Term structure of interest rates** is the relationship between the investment term and the interest rate
- The plot of this relationship is the **yield curve**



Bond valuation and the Yield curve

We can use the term structure to compute PVs of risk-free cash flows over different time horizons

Let r_n be the risk-free interest rate for an *n*-year term (this is also called spot interest rate). Then

$$PV = \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)^2} + \cdots + \frac{C_N}{(1+r_N)^N}$$

Computation

Bond pricing:

$$PV = \frac{cFV}{1+r_1} + \frac{cFV}{(1+r_2)^2} + \cdots + \frac{C_N}{(1+r_N)^N}$$

Yield to maturity

Definition

The Yield To Maturity (YTM) is the *constant, hypothetical discount rate* that, when used to compute the PV of a bond's cashflows, gives you the bond's market price as the answer.

Why is YTM used?

- Are the average return the investor receives from holding the bond.
- Yields are *annual* discount rates, they are much more easily compared than bond prices (prices can be different due to different coupon rates).

Computation

Assume you know the price of a coupon bond:

$$P = \underbrace{\frac{cFV}{y}\left(1 - \frac{1}{(1+y)^n}\right)}_{\text{This is 0 for zero-coupon bonds}} + \frac{FV}{(1+y)^n}$$

n-year annual coupon bond

y is the constant discount rate that solves these equations.

- *y* is the rate ***per coupon interval***. The yield Y is then quoted as a stated annual rate (APR).

Yield to maturity

Example

Consider a 1 year 12% coupon with **semi-annual** coupons and face value of €1000. If the market price of the bond is €1020, what is *Y*, i.e. the bond's yield to maturity?

1. First determine the coupon: $C = \frac{12\%}{2} \times 1000 = 60$
2. $1020 = \frac{60}{1+y} + \frac{1060}{(1+y)^2} \Leftrightarrow y \approx 4.9\%$

Note: The solution can be found using excel, a financial calculator, trial and error or in this specific case the quadratic formula.

Recall this is y for the coupon interval. Now we need to write is as annual rate (using the APR convention):

$$Y \approx 2 \times 4.9\% = 9.85\%$$

Yield to maturity as function of spot rates

What is the yield to maturity of a two-year 10% annual coupon bond when market rates (i.e., the yield curve) look as follows?

Year	Spot rate
1	4%
2	6%

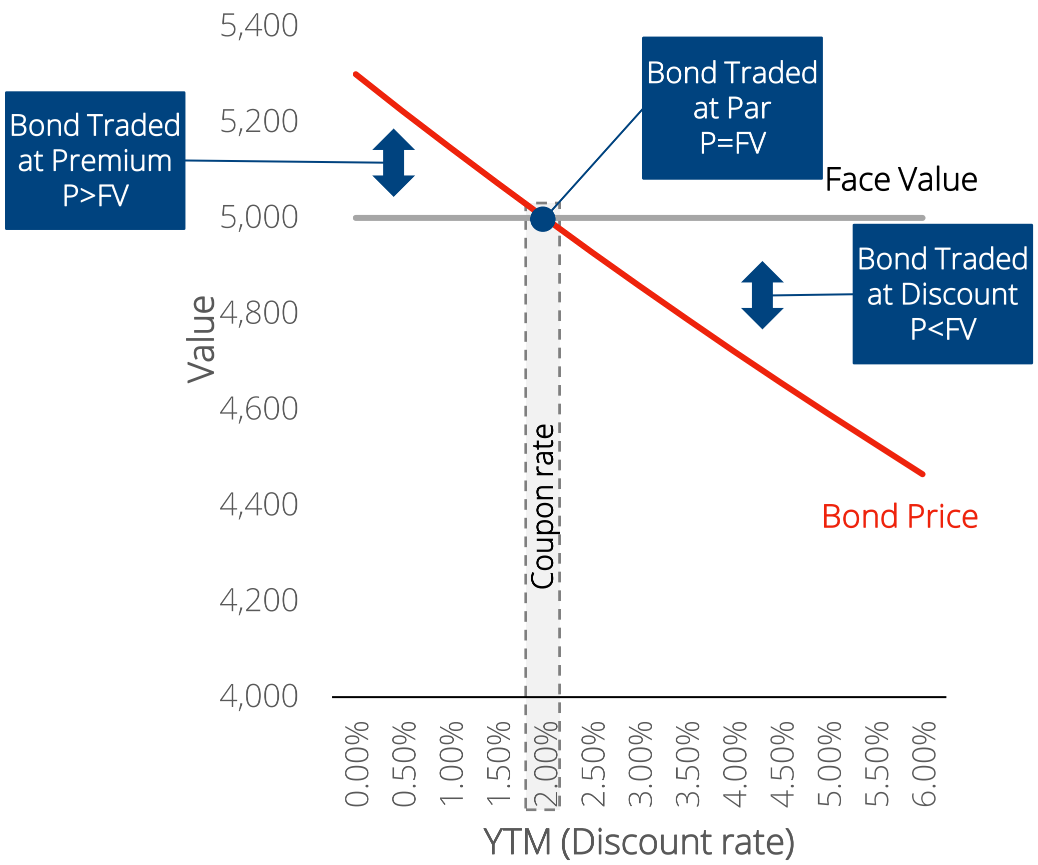
1. Compute bond price first: $P = \frac{10}{1.04} + \frac{110}{1.06^2} = 107.515$
2. Compute the yield to maturity: $107.515 = \frac{10}{1+y} + \frac{110}{(1+y)^2} \leftrightarrow y = 5.9\%$

The YTM is *a weighted average* of the spot rates with equal and shorter maturities!

Note: Bond prices can be computed using the YTM or the spot rates.

Yield to Maturity and Bond Prices

Bond price as a function of the YTM
(T=3Y, c=2%, FV=5000)



Summary

Relationship between Coupon rate and YTM		Relationship between Bond Price and Face Value	Bond Trading	Intuition
C > YTM	⇔	P > FV	Premium or Above Par	Every year the investor gets (coupon) more than required (YTM) and thus he is willing to pay a premium
C = YTM	⇔	P = FV	Par	Every year the investor gets (coupon) exactly the amount he requires (YTM)
C < YTM	⇔	P < FV	Discount or Below Par	Every year the investor gets (coupon) less than required (YTM) and thus he only buys the bond with a discount

Bond prices when YTM is constant

Expected return of a bond in a year:

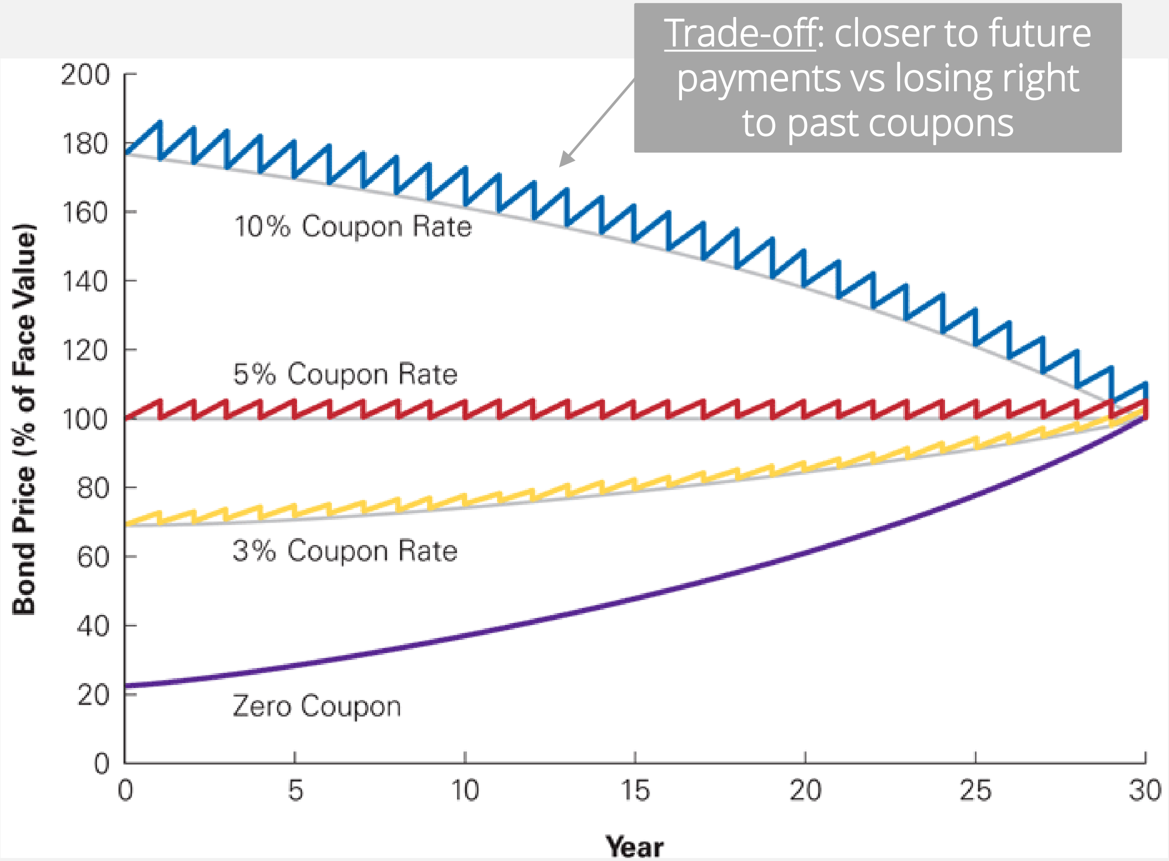
$$r = \underbrace{\frac{cFV}{P_0}}_{\text{Coupon yield}} + \underbrace{\frac{E_0\,[P_1 - P_0]}{P_0}}_{\text{Capital gain}}$$

- When YTM constant at 5%, investors earn a 5% return for each bond every year!

Example

Consider T=30y, annual coupons, c=10%, FV=100, YTM=5%, P_0=176.86 and P_1=175.71.

$$r = \frac{10}{176.86} + \frac{175.71 - 176.86}{176.86} = 5\%$$



Bonds - Duration

Dependence of bond prices on interest rates

- Fixed-income instruments are risky even if coupon and principal are guaranteed:
  - There is an inverse relation between yields and prices.
  - Interest rates fluctuate.
- Therefore, bond prices are sensitive to changes in interest rate.

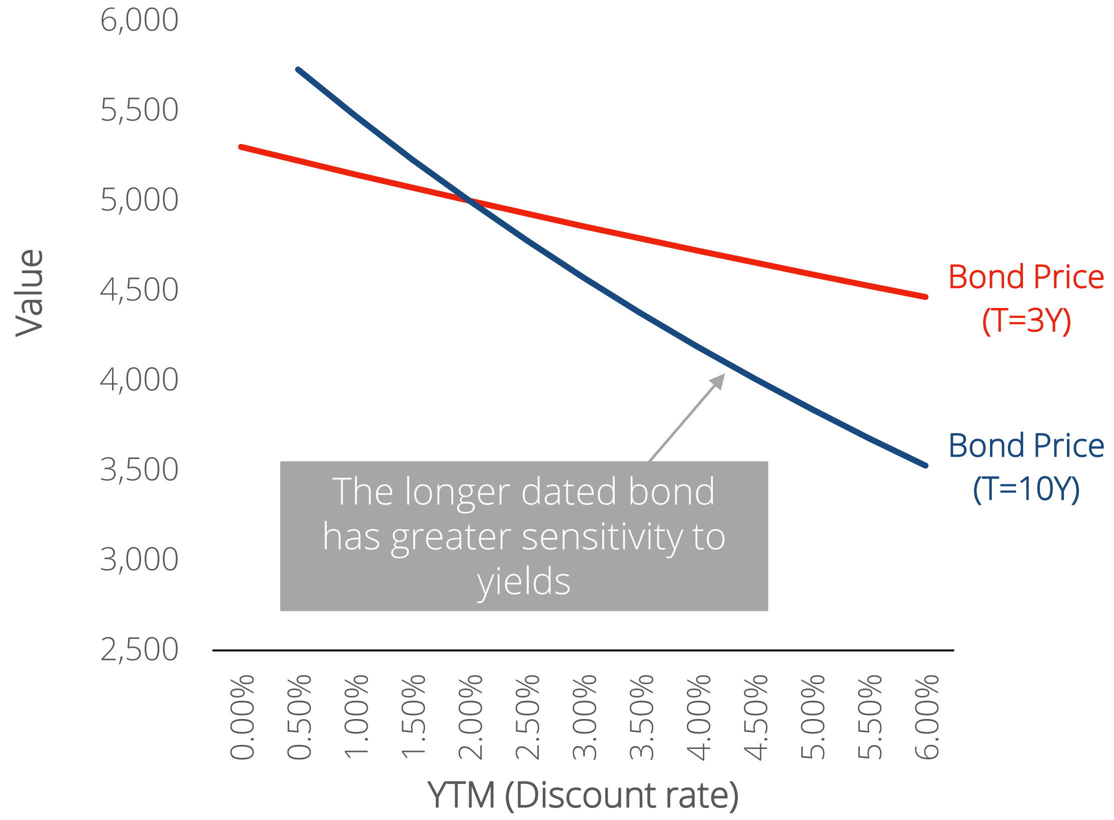
Facts:

1. *Prices and yields (or interest rates) are inversely related*
2. *Prices are convex* - the higher the convexity, the more sensitive the bond price is to decreasing interest rates and the less sensitive the bond price is to increasing rates

Dependence of bond prices on interest rates

Bond price as a function of the YTM

(c=2%, FV=5000, T=3Y OR T=10Y)



Duration

We want to measure the sensitivity of prices to yields as the percentage change in price for a 1% change in yields. This is the elasticity of the bond price w.r.t. yields. This is called the Macaulay Duration of the bond:

$$D = \frac{1}{P}\sum_{t=1}^{T} t \frac{C_t}{(1+y)^t}$$

- Intuition: It is computed by calculating the weighted time until each payment of the bond

Measuring the sensitivity of bond prices to interest rates

Duration

- Can be used to find approximate percentage changes in the price of a bond for a known change in yields:

$$\underbrace{\frac{\Delta P}{P}}_{\text{\% change in P}} \approx -\frac{D}{1+y} \underbrace{\Delta y}_{\text{Change in yields}}$$

- **Modified duration** measures percentage change in price per 1 percentage point change in yield:

$$D^M = \frac{D}{1+y}$$

Application

Example

Consider a 2-year bond with a 10% annual coupon and a yield of 4% annually compounded. What is the duration?

1. $\mathrm{P} = \frac{10}{(1+0.04)} + \frac{110}{1.04^2} = 9.615 + 101.70 = 111.32$
2. $D = \frac{1}{P} \sum_{t=1}^{T} t \frac{C_t}{(1+y)^t} =$

$$= \frac{1}{111.32}[1 * 9.615 + 2 * 101.7] = 1.92$$

And the Modified Duration is:

$$D^M = \frac{1.92}{1.04} = 1.84$$

Duration

$$D = \frac{1}{P}\sum_{t=1}^{T} t \frac{C_t}{(1+y)^t}$$

- Always between zero and the time to the final payment for the bond.
- For a zero coupon bond with T periods to maturity, the duration is always exactly T.

High duration

Bonds are **more** sensitive to interest rate changes:

- Long term bonds
- Low coupon bonds

Low duration

Bonds are **less** sensitive to interest rate changes:

- Short term bonds
- High coupon bonds

Higher sensitivity the larger share of cashflows is realized later

Lower sensitivity the larger share of cashflows is realized early

Key takeaways

01 **Value stocks using Dividend Discount Models.**

02 **Understand the tradeoffs between dividend payments and reinvestment in the firm. Computation of the Present Value of Growth Opportunities.**

03 **Bond valuation and its relationship with yield curve and yield to maturity.**

04 **Understand the sensitivity of bond prices to changes in interest rates. Application of the Duration formula.**