Video Lecture

Time Value of Money
Interest rates and Cash-flows

Advance Financial Management

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Interest rate

What is the appropriate interest rate?

- Real returns and inflation

- Interest rate quotes

Computation

- Should always use effective interest rates

- Consistency is key!
  - Discount nominal cash-flows with nominal interest rates and discount real cash-flows with real interest rates!

Real returns and inflation

- Need to distinguish between:
  - a nominal interest rate (the growth rate of money) and
  - a real interest rate

- Let π be the inflation rate.

$$1 + r_{nom} = (1 + r_{real})(1 + \pi)$$
$$= 1 + r_{real} + \pi + r_{real}\,\pi$$

$$r_{\mathrm{nom}} \approx r_{real} + \pi$$

- To convert a nominal cash-flow into a real cash-flow:

$CF_{real} = CF_{nom} / (1+\pi)$

Real returns and inflation

- Suppose that one year ago you deposited €1000 in a bank account with an interest rate of 10%.
- You are now about to collect €1100. Assume $\pi = 4\%$

Computation

- Real rate of return: $1.10 = (1+r_{real}) \times 1.04 \quad \Leftrightarrow \quad r_{real} = 1.10 / 1.04 - 1 = 5.77\%$
- What is the present value of this deposit one year ago?
  - discounting nominal cash-flow at nominal interest rate: 1100/1.10 = 1000
  - discounting real cash-flow at real interest rate:
    - $CF_{real} = 1100 / 1.04 = 1057.7$
    - At real interest rate → 1057.7 / 1.0577 = 1000

Quoting of interest rates

Interest rates are often stated as either:

- EARs – **effective anual rates**. This is the actual amount of interest earned at the end of one year after taking into consideration compounding.
- APRs - **annual proportional rate** or **stated annual rate**. This is the amount of simple interest earned at the end of one year.

Example

Suppose your bank pays compounded interest monthly at a rate of 0.5%. How does the bank state this as an annual rate?

- If stated as **EAR**: $1 + EAR = (1 + 0.005)^{12} \Leftrightarrow EAR = 1.005^{12} - 1 = 6.17\%$
- If stated as **APR**: $APR = 12 \times 0.005 = 6\%$.
  - In this case, the effect of compounding is ignored, even though the bank pays compounded interest.

The **EAR** is 6.17%

The **APR** is 6%, with monthly compounding

Quoting interest rates

Let *m* be the compounding frequency (e.g., week, month, quarter) per year, and r_m the effective interest rate in that period.

Generalizing for different compounding frequencies:

- APR = m x r_m

Typically, the quoted interest rate is the annual one and the effective rate for the compounding frequency is then:

$$r_m = \frac{APR}{m}$$

- EAR = $(1+\underbrace{APR/m}_{r_m})^m-1$

Example

Suppose you have a credit card that charges an interest rate of 18% (APR, compounded monthly). If you owed €100 on this account for two entire years what would your account balance be in two years?

Computation 1

Compute **EAR**: $EAR = (1+APR/m)^{m}-1$

$APR = 18\%$ and $m = 12$

$EAR = (1 + 18\%/12)^{12} – 1 = 19.56\%$

Use the EAR to compute the future value of €100 in two years

$FV = 100 \times (1.1956)^{2} = €142.95$

Example

Suppose you have a credit card that charges an interest rate of 18% (APR, compounded monthly). If you owed €100 on this account for two entire years what would your account balance be in two years?

Computation 2

Compute the effective monthly interest rate:

$r_m = 18\%/12 = 1.5\%$

Use this rate to compute the future value of €100 in two years

Number of months = 12 x 2 = 24 months

$FV = 100 \times (1+1.5\%)^{24} = €142.95$

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Special cash-flow Streams

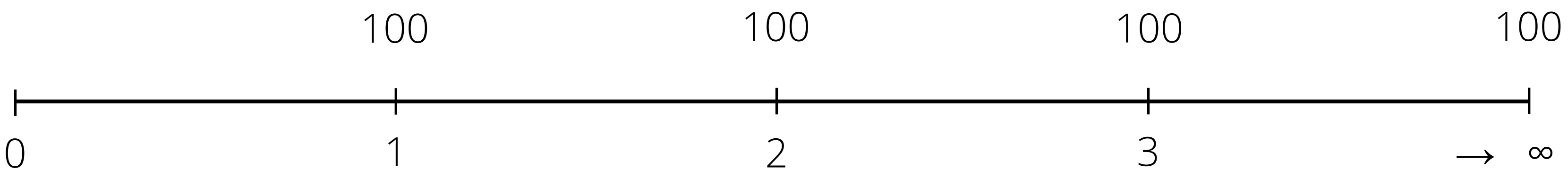
Special types of CF streams

Cash Flow		Future CF	Future CF	Future CF	Future CF	Future CF	Future CF
Time	0 (Now)	1 year from now	2 years from now	3 years from now	4 years from now	5 years from now	(...)

	With constant cash-flows	With cash-flows growing at a constant rate
Stream of perpetual cash-flows	Perpetuity	Growing perpetuity
Stream of cash-flows for a limited number of periods	Annuity	Growing annuity

Perpetuity

- Suppose you have the right to receive €100 per year, forever. What is this right worth?
- We need to compute the value of this entire stream of payments.
- We can do it one by one…

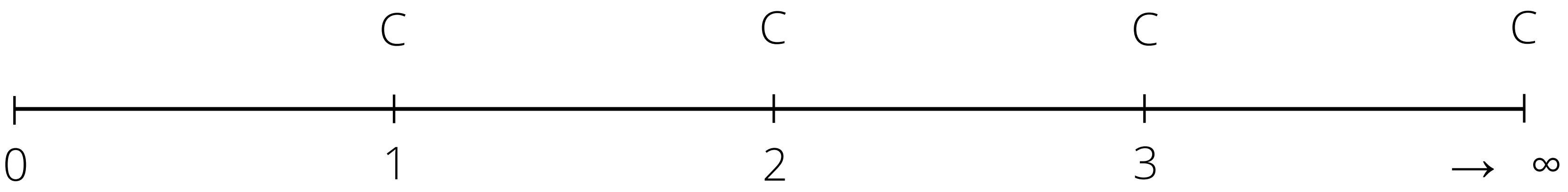


$$PV = \frac{100}{1+r} + \frac{100}{(1+r)^2} + \frac{100}{(1+r)^3} + \cdots = \sum_{n=1}^{\infty} \frac{100}{(1+r)^n}$$

- Or we can use the perpetuity formula!

Perpetuity

- Perpetuity is a stream of fixed nominal cash-flows C ***at the end of every period*** received forever.



- PV of perpetuity = $\frac{C_1}{r}$

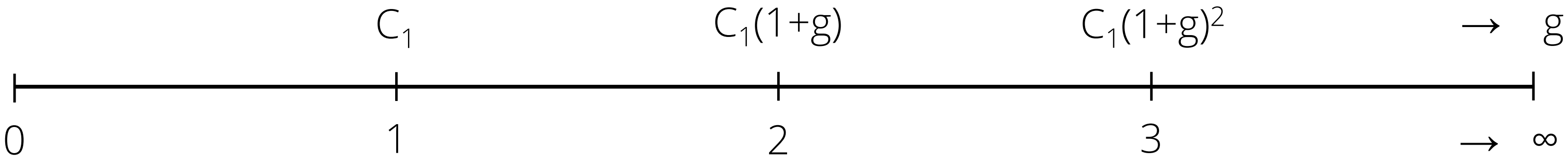
Computation

Assume the EAR is 4%. Then the right is worth:

$$PV = \frac{100}{0.04} = 2500$$

Perpetuity with growth

- Stream of cash-flows C that grows periodically at a rate g and that is received at the end of every period forever



- PV of perpetuity with growth = $\sum_{n=1}^{\infty} \frac{C_1(1+g)^{n-1}}{(1+r)^n}$

- PV of perpetuity with growth = $\frac{C_1}{r-g}$

Computation

You have the right to receive a growing cash-flow ***at the end of every year*** forever. The first cash-flow is €100 and grows at an annual rate g=2%. What are you willing to pay for this right if the EAR is 4%?

$$PV = \frac{100}{0.04-0.02} = 5000$$

Annuity

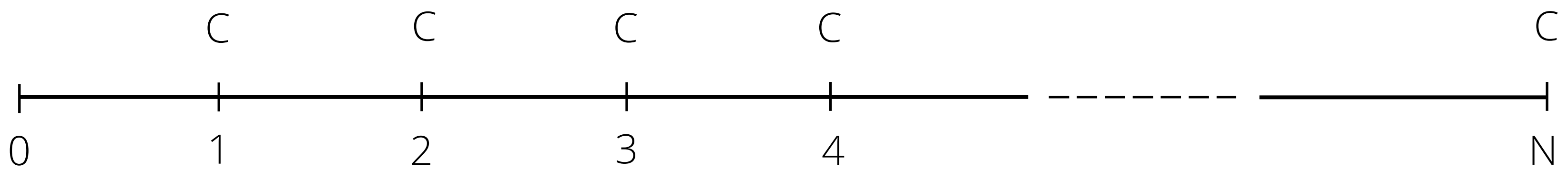
- Suppose you have the right to receive €100 per year, for 20 years. Are you willing to sell this right for €1,900?
- Assume an annual interest rate of r = 10%
- We can do it one by one.....

$$PV = \frac{100}{1.1} + \frac{100}{1.1^2} + \cdots + \frac{100}{1.1^{20}}$$

- Or, we can use the *annuity formula*!

Annuity

- These are more common cashflow streams, consisting of the payment of a fixed nominal cashflow, once per period but only for a known, finite number of periods.



- PV of annuity = $\sum_{n=1}^{N} \frac{C_1}{(1+r)^n}$
- PV of annuity = $\frac{C_1}{r}\left(1 - \frac{1}{(1+r)^N}\right)$

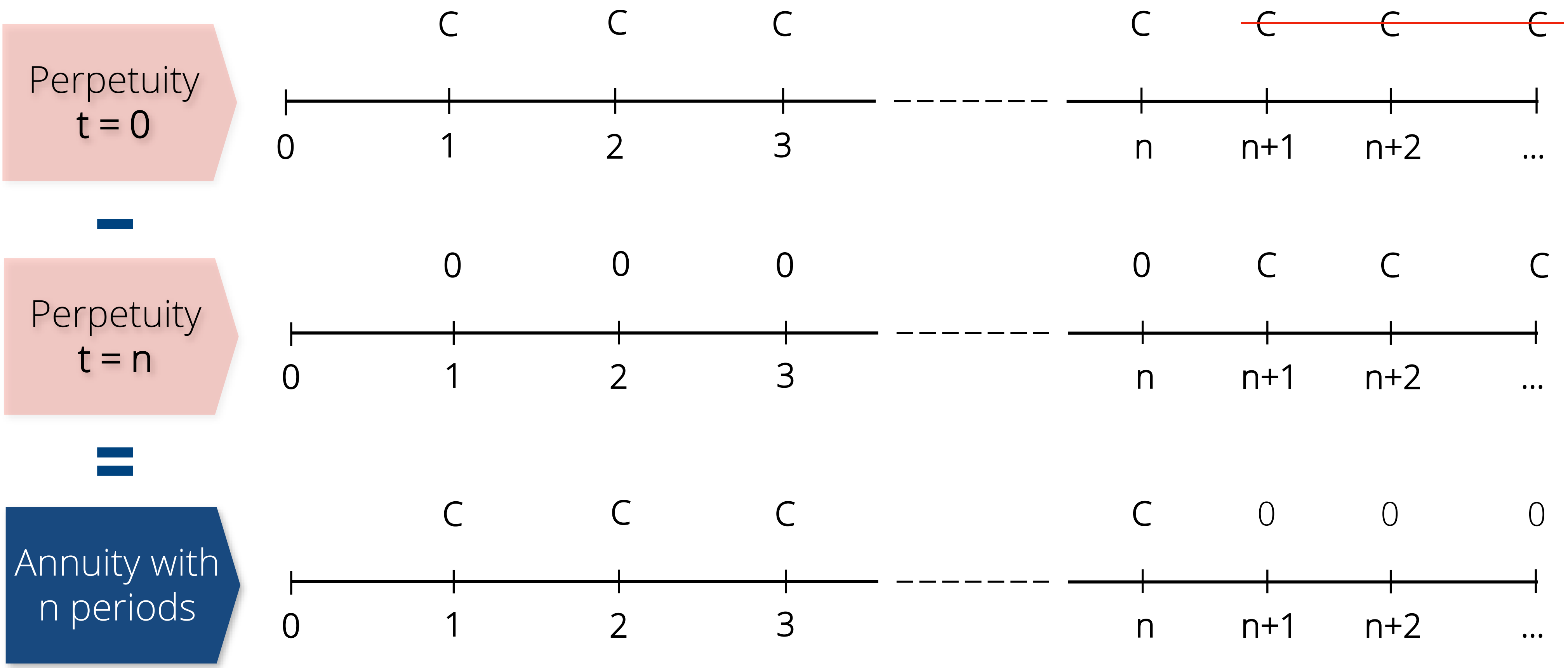
Computation

The value of the right in the example: $PV = \frac{100}{0.1}\left(1 - \frac{1}{1.1^{20}}\right) = 851.4$.

We would be willing to sell this right for €1900.

Annuity

Annuity formula is related to the perpetuity formula: we can obtain an annuity by subtracting two perpetuities:



Annuity

Annuity formula is related to the perpetuity formula: we can obtain an annuity by subtracting two perpetuities:

$$PV(perp.t = 0) = \frac{C_1}{r}$$

$$-$$

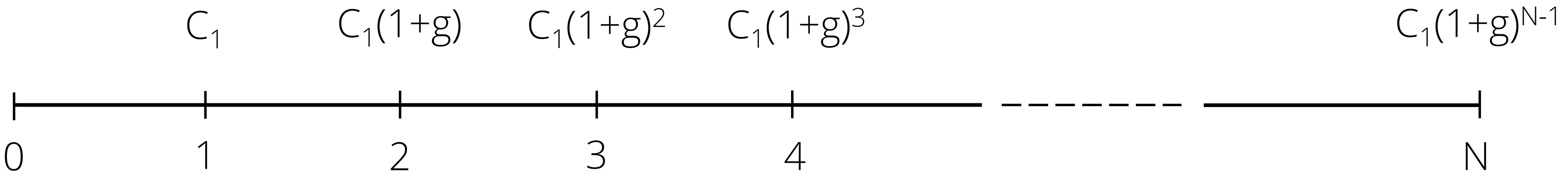
$$PV(perp.t = n) = \frac{C_1}{r}\boxed{\frac{1}{(1+r)^n}}$$

$$=$$

$$PV(annuity\; n\; periods) = \frac{C_1}{r} - \frac{C_1}{r}\frac{1}{(1+r)^n} = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

Annuity with growth

- Stream of cash-flows C that grows periodically at a rate g and that is received ***at the end of every period*** for N periods, discounted at rate r



- PV of annuity with growth = $\sum_{n=1}^{N} \frac{C_1 (1+g)^{n-1}}{(1+r)^n}$
- PV of annuity with growth = $\frac{C_1}{r-g}\left(1-\left(\frac{1+g}{1+r}\right)^N\right)$

Computation

Suppose you have the right to receive 20 payments ***at the end of every year***. The first will be €100 and it will then grow at an annual rate of 2%. How much is this worth if the EAR is 10%?

PV = $\frac{100}{0.1-0.02}\left(1-\left(\frac{1.02}{1.1}\right)^{20}\right) = 973.90$

How to apply the formulas correctly

Computation

- Formulas assume the first cash-flow accrues <u>at the end</u> of the first year!
    - Remember to apply the formulas taking this into account
- Use effective rates
- **Consistency is key!**
    - Frequency of payments, discount rates and growth rates should be the same (e.g. with monthly payments, use monthly discount and growth rates).
        - **Note:** the frequency of payments dictates what rates you should use
    - Discount nominal cash-flows with nominal interest rates and discount real cash-flows with real interest rates!

Key takeaways

01 **Understand the concept of the discount rate or opportunity cost of capital.**

02 **Value a stream of cash flows, either using the future value or the present value.**

03 **Apply the perpetuity and annuity formulas.**

04 **Convert quoted interest rates in the form provided into effective interest rates which are appropriate to use in the valuation of a stream of cash flows.**