Video Lecture

# Time Value of Money

Advance Financial Management

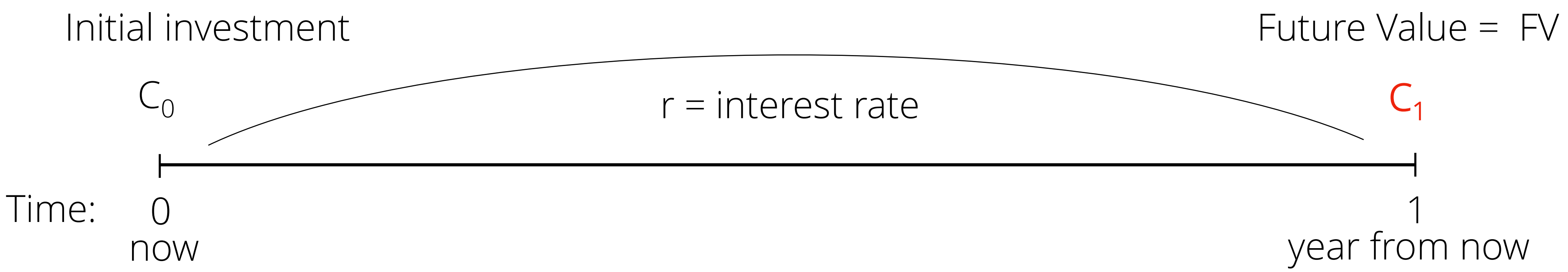
Margarida Soares & Fábio Soares Santos

# Overview

- Future Value and Present Values
- Present Value rules
- Interest rate
- Special cash-flow Streams

# Future Value – one time period

Timeline:




*Computation*

$FV_1 = C_1$ = Initial investment + interest $= C_0 + r \times C_0 = C_0\,(1 + r)$

**Example:** $C_0$=100 and r=4%. Then $C_1$=100 x 1.04 = 104

# Future Value – multiple time periods

Initial investment — Future Value = FV

$C_0$ — $C_2$

Time: 0 now — 1 year from now — 2 years from now

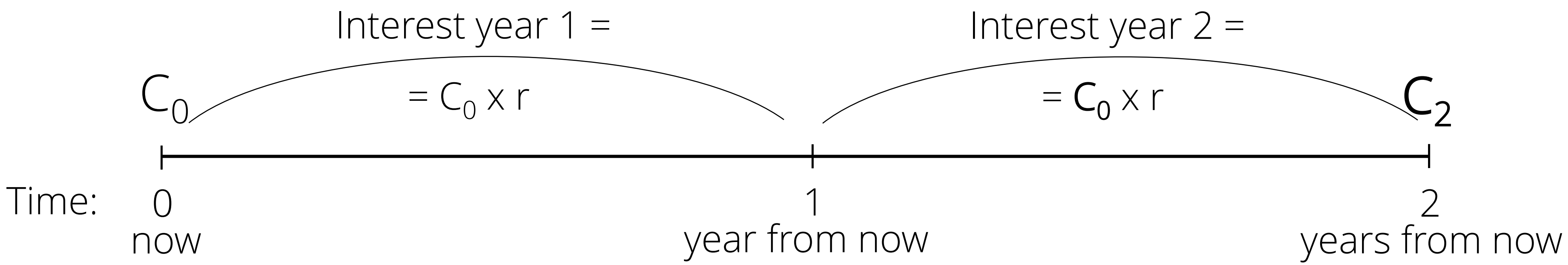
*Computation*

$FV_2 = C_2 =$ Initial investment + **interest**

What the **interest** is depends on whether you earn:

- Simple interest
- Compounded interest

# Future Value – multiple time periods, simple interest



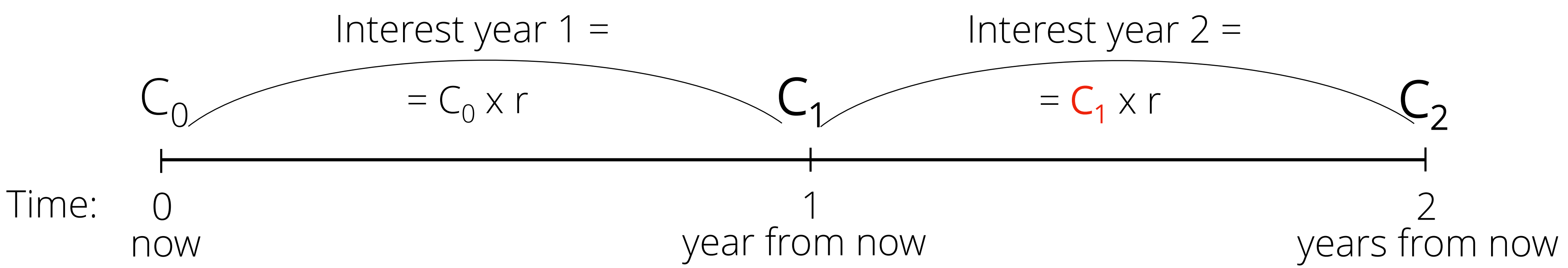

*Computation*

*Simple interest* only pays interest on the original investment

$FV_2 = C_2 = C_0 + r \times C_0 + r \times \mathbf{C_0} = C_0 + 2\, r\, C_0$

Example: $FV_2 = 100 + 2 \times 0.04 \times \mathbf{100} = 108$

# Future Value – multiple time periods, compounded interest




## *Computation*

*Compounded interest* pays interest on the original investment and also on the accumulated interest

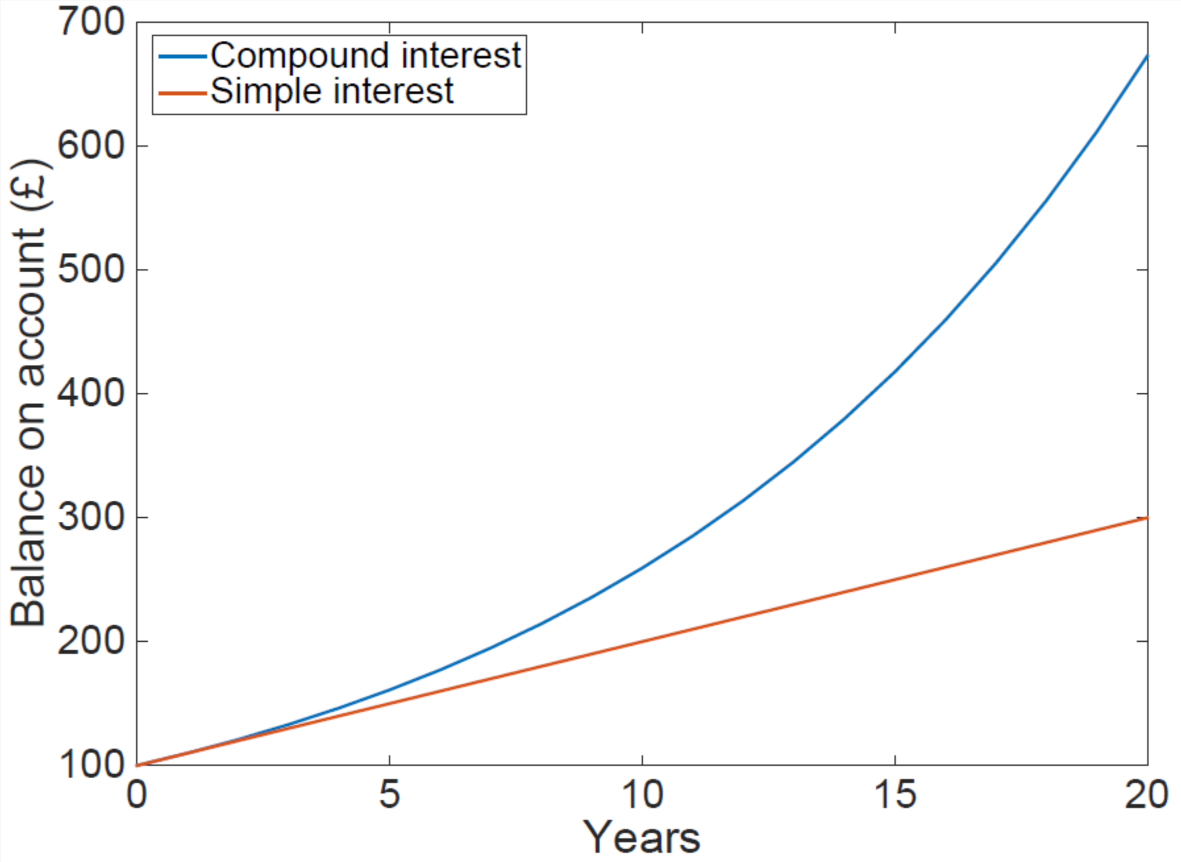
$FV_2 = C_2 = C_1 + r \times C_1 = (1 + r)\, C_0 + r \times (1 + r)\, C_0 = C_0\, (1 + r)^2$

Generalizing for n periods: $FV_n = C_0\, (1+r)^n$

Example: $FV_2 = 100\, (1.04)^2 = 108.16$

# Compounding effect

- With simple interest the amount of money invested increases linearly
- With compounded interest the amount invested increases exponentially



# Present Values

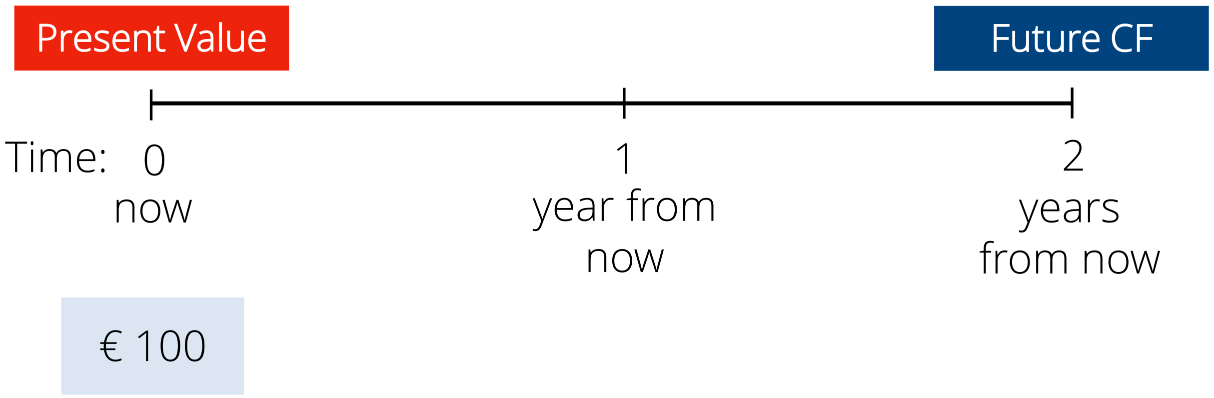
- Example: You have two alternative investments:
  1. pays €100 today or
  2. pays €100 in 2 years

  Which do you prefer?

How much are you willing to pay for each alternative?

*Computation*

- Alternative 1: €100 = $PV_1$



# Present Values

- Example: You have two alternative investments:
  1. pays €100 today or
  2. pays €100 in 2 years

  Which do you prefer?

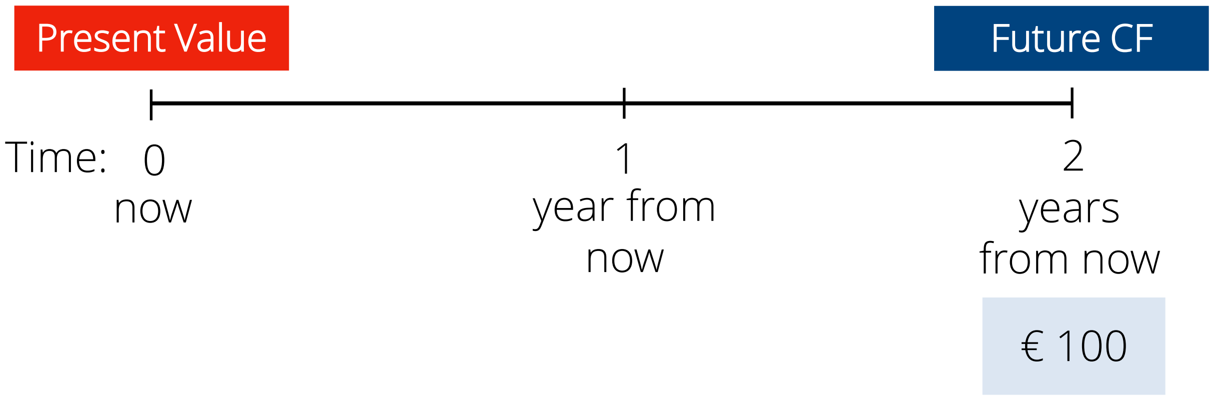
How much are you willing to pay for each alternative?

*Computation*

- Alternative 2: $PV_2$ = how much would you have to have deposited **today** to earn €100 in 2 years?

$FV_2 = D\,(1+r)^2 = €100 \quad \Leftrightarrow \quad D = €100\,/\,(1+r)^2$

$PV_2 = D = €100\,/\,(1+r)^2$.



# Present Values

- PV (Alternative 1) > PV (Alternative 2)

**Example**: if r = 4%. $PV_1$ = €100 and $PV_2$ = €100 / ($1.04^2$) = € 92.46

*Computation*

Generalizing for n periods: PV = $\frac{CF_n}{(1+r)^n}$

Advanced Financial Management | Time Value of Money

# Present Value Rules

# Rules of time travel

1. Only cash-flows at the same point in time can be compared or combined
2. To move a cash-flow forward in time, you must compound it:

   $FV_n = C_0 \times (1+r)^n$

3. To move a cash-flow backward in time, you must discount it:

   $PV = C_n / (1+r)^n$

# Compare cashflows at same point in time

- Which alternative do you prefer if the one year interest rate r = 4%?
  1. €100 today
  2. €103 in one year

*Computation*

- Compare cashflows (CF) at the same point in time (rule #1):
  - Compare the PV of both options: PV(1) = €100; PV(2) = €103/1.04 = €99 ⇒ Option 1 is better
  - Compare the FV of both options: FV(1) = €100x1.04 = €104; FV(2) = €103 ⇒ Option 1 is better
- In both cases option 1 is better. Comparing PV and FV is equivalent.

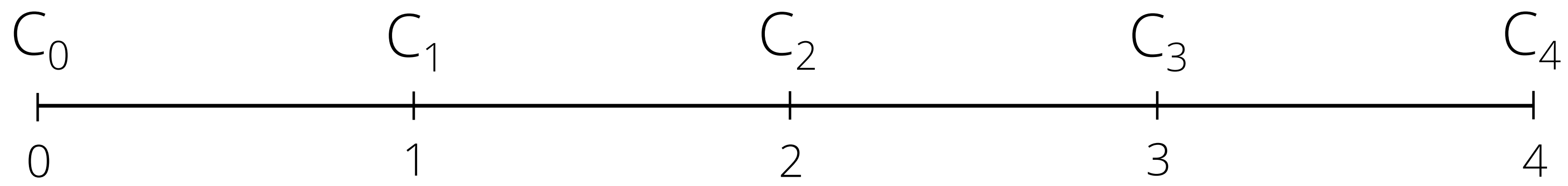
# Two main components of PV

- Cashflows – it matters what they are and *when* they occur
- Discount rate:
  - A discount rate is the reward that investors demand for accepting delayed rather than immediate gratification.
  - If you lend someone money for a year, you demand interest as you cannot instantly spend the money you have lent on consumption goods.
  - The discount rate is also called opportunity cost of capital because it is the return foregone by investing in a capital project rather than investing in freely-available securities.
  - Other names: interest rate, required rate of return or opportunity cost of capital.

- Note: Also used in this context is the term *discount factor* = $\frac{1}{1+r}$

# Valuing a stream of CFs

Most investment opportunities have multiple CFs which occur at different points in time




How do we compute the value today of the investment opportunity?

*Computation*

$$PV = PV(C_0) + PV(C_1) + PV(C_2) + PV(C_3) + PV(C_4) =$$

$$= C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \frac{C_4}{(1+r)^4}$$

Generalizing for n periods: PV = $\sum_{n=0}^{N} \frac{C_n}{(1+r)^n}$

# Project valuation using Net Present Value

Consider the following investment opportunity:

- Invest €100 today
- Receive €30 at the end of year 1
- Receive €75 at the end of year 2
- The interest rate is 4%

Should we make this investment?

- Not simply a €5 profit; cash-flows are not obtained at the same date!
- To answer this question, we compute the present value of all cash-flows:

*Computation*

*Net Present Value (NPV) = PV(benefits) – PV(costs)*

- PV (benefits) = 30 / 1.04 + 75 / $1.04^2$ = 98.1
- PV (costs) = 100
- NPV = -100 + 98.1 = -1.9 → *do not invest*!

# NPV: usage and implications

NPV rule:

- If NPV is positive, you should invest in the project
- If NPV is negative, you should turn down the investment opportunity

Comments:

- The discount rate used in the NPV calculation should reflect the project’s risk
- If you don’t know the cashflows associated with the project precisely, use the expected value of each cashflow instead.