Time Value of Money

Advanced Financial Management

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Key takeaways

01 **Understand the concept of the discount rate or opportunity cost of capital.**

02 **Value a stream of cash flows, either using the future value or the present value.**

Time value of money

- Reward demanded by investors for having an amount of money tied up in an investment

Why is it usually the interest rate?

- Interest rate is your opportunity cost.

- If you do not invest in a project or asset, then you could always deposit it in a bank and earn the market interest rate.

Compounded interest

- *Simple interest* only pays interest on the initial investment.

- ***Compounded interest*** pays interest on the original investment and also on the accumulated interest.

Example

Suppose the semi-annual interest rate r_s is 3%. If I invest €1 today, how much will I have earned at the end of 1 year (assuming compounded interest)?

$$1 + r_s + (1 + r_s) \times r_s = (1 + r_s)^2 = 1.0609$$

What was the return on your investment you *effectively* earned? 6.09%

This is called the Effective Annual Rate = EAR: $(1 + r_s)^2 = 1 + EAR \Leftrightarrow (1 + 3\%)^2 = 1 + EAR \Leftrightarrow r_s = 6.09\%$

Rules of time travel

1. Only cash-flows at the same point in time can be compared or combined
2. To move a cash-flow forward in time, you must compound it:

   $FV_n = C_0 \times (1+r)^n$
3. To move a cash-flow backward in time, you must discount it:

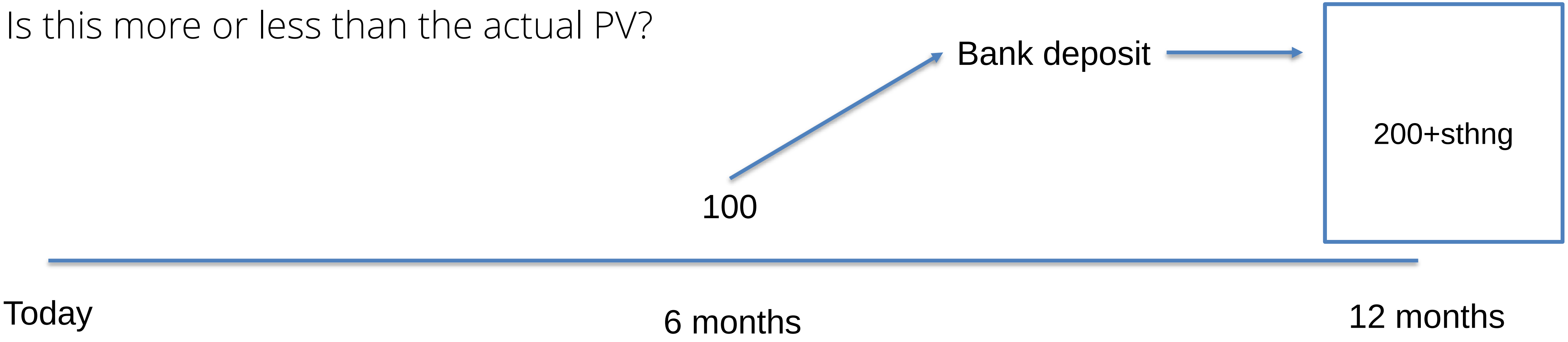
   $PV = C_n / (1+r)^n$

Frequency of payments: example

You receive semi-annual payments of €100 for 1 year. The EAR is 10%.

Is the PV = $\frac{200}{1.1} = 181.81$ correct?

No! Is this more or less than the actual PV?



$$PV = \frac{200+sthng}{1.1} > \frac{200}{1.1} = 181.81$$

We are going to see three ways of computing the PV

Frequency of payments: example

You receive semi-annual payments of €100 for 1 year. The Effective Annual Rate is 10%.

Computation

Note that 6 months are ½ of a year:

$$PV = \frac{100}{1.1^{1/2}} + \frac{100}{1.1^{1}} = 186.26$$

CAREFUL: This method only works if we have the Effective Annual Rate

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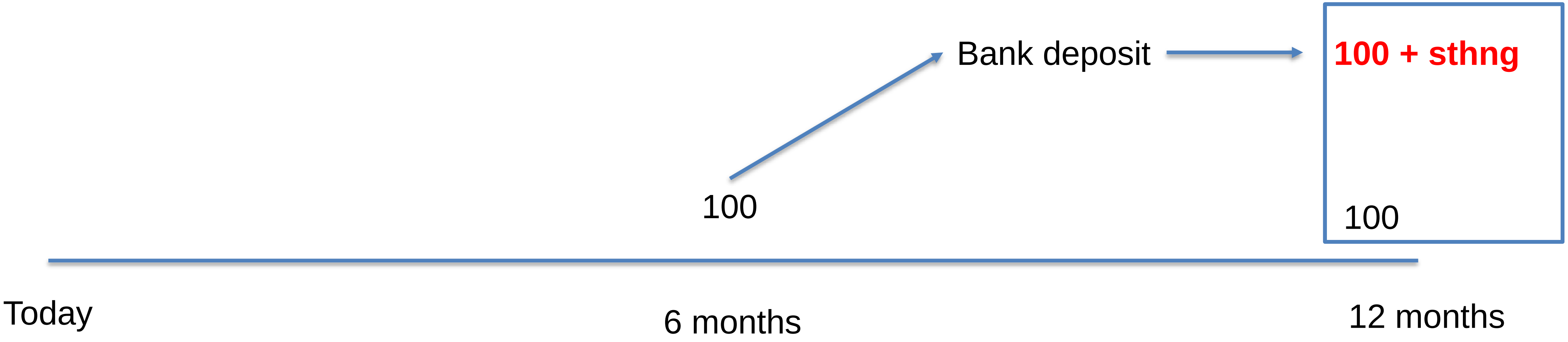

Frequency of payments: example

You receive semi-annual payments of €100 for 1 year. The Effective Annual Rate is 10%.

Computation

We can compute sthng as the interest payment of a 6-month deposit

$$100 + sthng = 100 \times 1.1^{1/2} = 104.88088 \Rightarrow PV = \frac{100 + 104.88}{1.1} = 186.26$$

CAREFUL: This method only works if we have the Effective Annual Rate

Frequency of payments: example

You receive semi-annual payments of €100 for 1 year. The EAR is 10%.

Computation

1. Compute the effective semi-annual rate:

$$(1 + r_s)^2 = 1 + EAR$$

$$(1 + r_s)^2 = 1 + 10\% \Leftrightarrow 1 + r_s = (1 + 10\%)^{1/2} \Leftrightarrow r_s = 4.9\%$$

2. Compute PV of cash-flows:

$$PV = \frac{100}{1.049} + \frac{100}{1.049^2} = 186.26$$

Present Value and Future Value: other examples

How long until you earn a certain amount?

If we deposit €5000 today in an account paying 10% per year, how many years will it take to grow to €10,000?

$FV_T = C_0(1+r)^T \Leftrightarrow 10{,}000 = 5000(1.1)^T$

$\Leftrightarrow 2 = 1.1^T$

$\Leftrightarrow \ln(2) = T \ln(1.1)$

$\Leftrightarrow T = \frac{\ln(2)}{\ln(1.1)} = 7.27$

$C_0 = Inital\ Investment \quad FV_T = Future\ Value\ at\ T$

$Divide\ both\ sides\ \ by\ 5{,}000$

When we have an unknown in the exponent logs are a useful tool because:

$$\ln(a^x) = x ln(a)$$

$Divide\ both\ sides\ \ by\ \ln(1.1)$

Present Value and Future Value: other examples

What rate do you need to earn a certain amount?

Assume total cost of college education will be €50000 in 12 years. You have €5000 to invest today. What rate of interest must you earn to cover the cost?

$FV_T = C_0(1+r)^T \Leftrightarrow 50{,}000 = 5{,}000\,(1+r)^{12}$

$\Leftrightarrow 10 = (1+r)^{12}$

$\Leftrightarrow 10^{1/12} = 1+r$

$\Leftrightarrow r = 21.15\%$

$C_0 = Inital\ Investment\ \ FV_{12} = Future\ Value\ at\ T = 12$

$Divide\ both\ sides\ by\ 5{,}000$

$We\ raise\ both\ sides\ to\ the\ power\ of\ \frac{1}{12}$

$Remember{:}\ \ (x^c)^d = x^{cd} \Rightarrow (x^{12})^{\frac{1}{12}} = x^1 = x$

Net present value

- Net present value is the present value of all positive cashflows minus the present value of all negative cashflows
- We never start projects with negative NPV
- If we have to choose a project, we always choose the highest NPV project

Exercise 1

Consider you are offered a project that requires 1000€ today and generates a cashflow of 500€ in one year and 600€ in two years. The rate you can earn in your deposits is 8%.

a. Should you undertake the project?
b. Consider the cashflows for year one and two are the same as before but you are allowed to make the 1,000€ investment at the end of the first year. Should you undertake the project?

Exercise 2

Consider the EAR is 7% and you are offered two projects:

Project A: generates a cashflow of 1000 every year for the next 30 years

Project B: generates a cashflow of 450 every 6 months for the next 30 years

The initial investment is 100 for both projects. Which project would you choose?