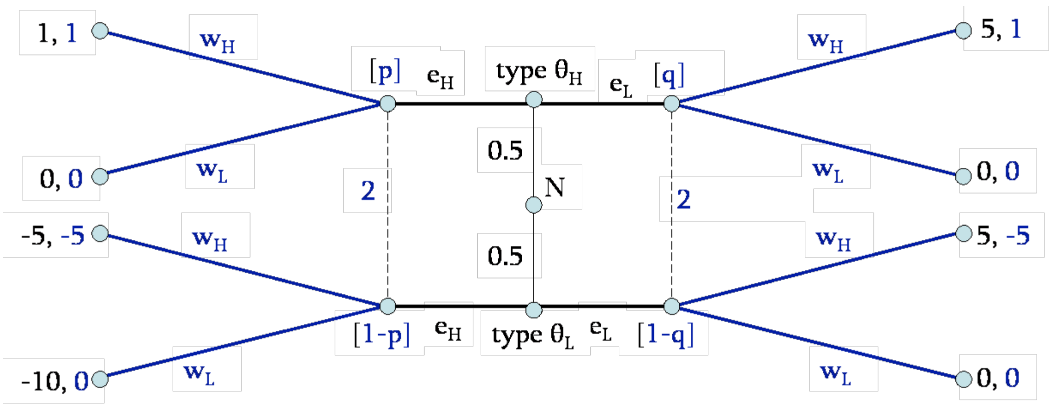
Assume that education is completely irrelevant for a worker's productivity. However, education may be used to signal a worker's productivity.
Assume that there are only two possible productivity levels: high (θ_H=6) and low (θ_L=0). Assume also that there are only two available education levels: high (e_H) and low (e_L). The cost of low education is 0 for both types but the cost of achieving a high education level is 10 for low-productivity agents and 4 for high-productivity agents.
Assume also that a firm can only set two possible wage levels: high (w_H=5) and low (w_L=0).

A high-productivity worker has an outside option and she will always refuse an offer of a low wage, in which case both the firm and the worker get a final payoff of 0 (this payoff already incorporates the education cost for the worker i.e. the worker's final payoff is 0 regardless of the education level).
For all other cases, an agent's payoff is her wage minus the education cost. An employer's payoff is the productivity minus the wage.

The timing of the game is as follows: Nature moves first, determining the productivity of an agent (where the probability of each type is 50%). Then, an agent observes her type and decides upon the level of education she will achieve. The employer observes the education level but does not observe the agent's type and sets a high or low wage.

a. Represent the game in extensive form
b. Find all separating Perfect Bayesian Equilibria of this game.
c. Is there a pooling PBE in which both worker types choose a high education level?

a.



b. First candidate: (θ_H chooses e_H; θ_L chooses e_L).
Beliefs: p=1, q=0.
Player 2's best response is w_H after e_H (1>0) and w_L after e_L (0>-5)
Player 1 of type θ_H does choose e_H (1>0); Player 1 of type θ_L does choose e_L (0>-5).
PBE: ((θ_H chooses e_H; θ_L chooses e_L), (w_H after e_H, w_L after e_L), p=1, q=0)
Second candidate: (θ_H chooses e_L; θ_L chooses e_H).
Beliefs: p=0, q=1.
Player 2's best response is w_L after e_H (1>0) and w_H after e_L (0>-5)
Player 1 of type θ_H does choose e_L (5>0); but Player 1 of type θ_L chooses e_L (5>-10). No PBE.

c. Pooling candidate: (θ_H chooses e_H; θ_L chooses e_H).
Beliefs: p=0,5, q in [0,1].
Player 2's best response is w_L after e_H (0>0,5*1+0,5*(-5)).
But then Player 1 of type θ_L does not choose e_H (5>-10 and 0>-10). No PBE.