# static games with complete information

## introduction:

examples

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## equilibrium in dominant strategies

dominant and dominated strategies:

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definition of Nash equilibrium

interpretations

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# example:
# prisoner's dilemma

| 1 \ 2 | Not to confess | Confess |
|---|---|---|
| Not to confess | -1,-1 | -6,0 |
| Confess | 0,-6 | -3,-3 |

# example: coordination game

| 1 \ 2 | Book launch | Movie |
| --- | --- | --- |
| Book launch | 2,1 | 0,0 |
| Movie | 0,0 | 1,2 |

# example: matching pennies

| 1 \ 2 | Heads | Tails |
| --- | --- | --- |
| Heads | -1,+1 | +1,-1 |
| Tails | +1,-1 | -1,+1 |

# definition of a game

Game:

- Set of players {1,2,3,…,N}

- Set of (pure) strategies for each player $S_i$ (i=1,…,N)

- Payoff function for each player: $\Pi_i(s_i,s_{-i})$ (i=1,…,N)

# dominant and dominated strategies (strong definition)

Def: $s_i$ is a strictly dominant strategy for player i or strongly dominates all other strategies for player i

if, for all $s_i' \in S_i$ such that $s_i' \neq s_i$, and for all $s_{-i} \in S_{-i}$,

$\Pi_i(s_i, s_{-i}) > \Pi_i(s_i', s_{-i})$.

(and $s_i'$ is a strictly dominated strategy for player i)

# dominant and dominated strategies (weak definition)

Def: $s_i$ is a weakly dominant strategy for player i or weakly dominates all other strategies for player i if,

for all $s_i' \epsilon S_i$ such that $s_i' \neq s_i$,

for all $s_{-i} \epsilon S_{-i}$, $\Pi_i(s_i, s_{-i}) \geq \Pi_i(s_i', s_{-i})$

and for some $s_{-i} \epsilon S_{-i}$, $\Pi_i(s_i, s_{-i}) > \Pi_i(s_i', s_{-i})$

(and $s_i'$ is a weakly dominated strategy for player i)

# example:
# weakly dominant strategy

| 1 \ 2 | L | R |
|---|---|---|
| U | 7,3 | 5,3 |
| D | 7,0 | 3,-1 |

# equilibrium in dominant strategies
# dominant strategy solution

Def: A game has a dominant strategy solution if every player has a dominant strategy.

Def: $(s_1,\ldots,s_N)$ is an equilibrium in dominant strategies if for all $i=1,\ldots,N$, $s_i$ is a dominant strategy.

# Nash equilibrium (definition)

Def: Strategy $s_i^*$ is a best response to $s_{-i}^* \epsilon S_{-i}$
if $\Pi_i(s_i^*,s_{-i}^*) \geq \Pi_i(s_i,s_{-i}^*)$ for all $s_i$

Def: A vector of strategies $s^*=(s_1^*,s_2^*,\ldots,s_N^*)$ is
a Nash equilibrium
if $\Pi_i(s_i^*,s_{-i}^*) \geq \Pi_i(s_i,s_{-i}^*)$ for all $s_i$ and for all $i$

# Nash equilibrium (interpretations)

- play prescription
- preplay communication
- rational introspection
- focal point
- trial and error

# example: prisoner's dilemma

| 1 \ 2 | Not to confess | Confess |
|---|---|---|
| Not to confess | -1,-1 | -6,0 |
| Confess | 0,-6 | -3,-3 |

# example: coordination game
# problem: uniqueness

| 1 \ 2 | Book launch | Movie |
|---|---|---|
| Book launch | 2,1 | 0,0 |
| Movie | 0,0 | 1,2 |

# example: matching pennies
# problem: existence

| 1 \ 2 | Heads | Tails |
|---|---|---|
| Heads | -1,+1 | +1,-1 |
| Tails | +1,-1 | -1,+1 |

# mixed strategies
## definition

Def: Suppose a player has M pure strategies $s^1, s^2, \ldots, s^M$.
A mixed strategy for this player is a probability distribution over her pure strategies $(p^1, p^2, \ldots, p^M)$ where $p^k \geq 0$ for all k, and $\Sigma_k p^k = 1$.

Note: A pure strategy is also a mixed strategy.

Def: The support of a mixed strategy is the set of pure strategies to which the mixed strategy attributes a positive probability.

# mixed strategies

## example: coordination game

| 1 \ 2 | | q | 1-q |
|---|---|---|---|
| | | Book launch | Movie |
| p | Book launch | 2,1 | 0,0 |
| 1-p | Movie | 0,0 | 1,2 |

# mixed strategies

## expected payoff

Example:

player 1's expected payoff in the coordination game is

p[ 2.q+0.(1-q)]+(1-p)[0.q+1.(1-q)]

General case:

- each player has M pure strategies $s_i^1, s_i^2, \ldots, s_i^M$ and plays a mixed strategy $(p_i^1, p_i^2, \ldots, p_i^M)$ where $p_i^k \geq 0$ for all k, and $\Sigma_k p_i^k = 1$.
- player i's expected payoff is $\Sigma_j \Sigma_k p_i^j . p_{-i}^k \, \Pi_i(s_i^j, s_{-i}^k)$

# mixed strategies

## example: best response

| 1 \ 2 | L | M1 | M2 | R |
|---|---|---|---|---|
| U | 1,0 | 4,2 | 2,4 | 3,1 |
| ~~M~~ | ~~2,4~~ | ~~2,1~~ | ~~2,2~~ | ~~2,1~~ |
| D | 4,2 | 1,4 | 2,0 | 3,1 |

# mixed strategies

## best response

The expected payoff to a mixed strategy is simply an average of the payoffs of the pure strategies in the support of this mixed strategy.

A mixed strategy $(p^1, p^2, \ldots, p^M)$ is a best response to $s_{-i}$ if and only if each of the pure strategies in its support is itself a best response to $s_{-i}$.

In that case, any mixed strategy over that support will be a best response.

# mixed strategies and domination

- Mixed strategies may dominate some pure strategies.
- Adding mixed strategies has no impact on dominant strategy equilibria:

  if there is a pure strategy that dominates every other pure strategy, it will also dominate every other mixed strategy;

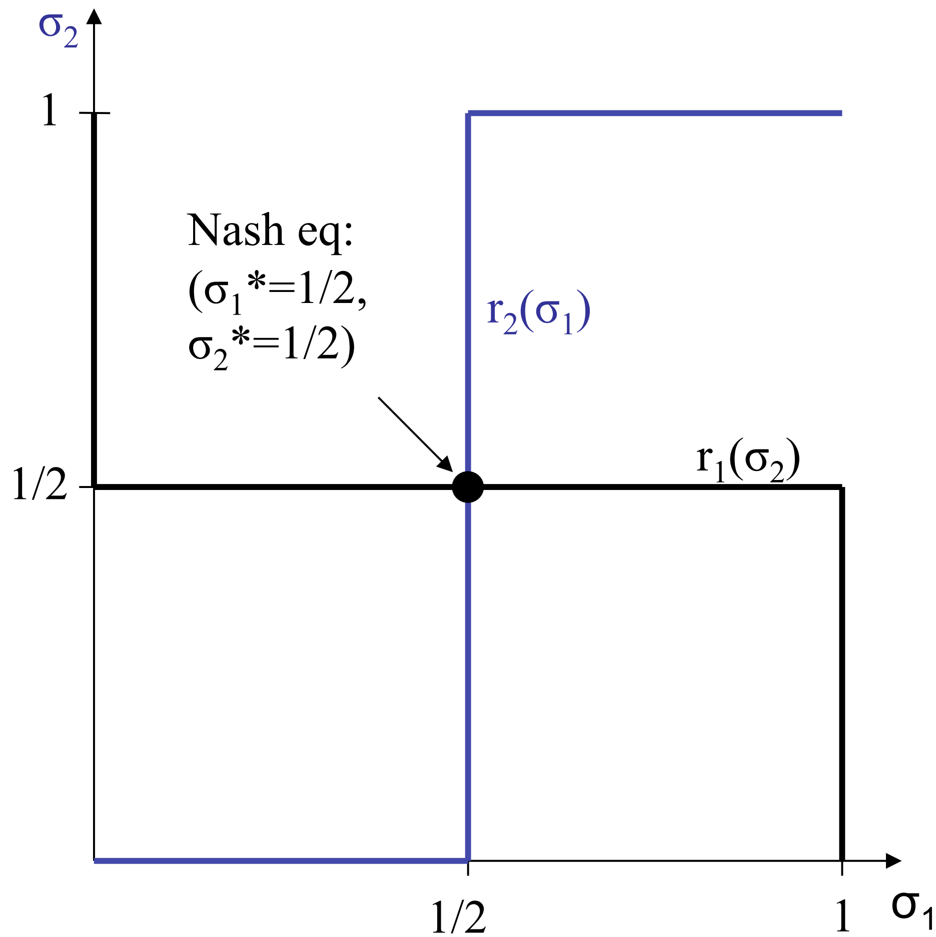
  if there is no dominant strategy in pure strategies, there cannot be one in mixed strategies either.

# mixed strategies

## example: Nash equilibrium

| | 1 \ 2 | Heads (q) | Tails (1-q) |
|---|---|---|---|
| p | Heads | -1,+1 | +1,-1 |
| 1-p | Tails | +1,-1 | -1,+1 |

# mixed strategies
## example: Nash equilibrium



# mixed strategies and Nash equilibrium

- In every game there is always a Nash equilibrium in mixed strategies.
- Examples: matching pennies, coordination game

- Are mixed strategies reasonable?